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AN EMPIRICAL ENQUIRY ON THE SHORT-RUN
DYNAMICS OF OUTPUT AND PRICES

Roque B. Fernandez ^{*/}

^{*/} Latin American Institute for Economic and Social Planning

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UNA INDAGACION EMPIRICA SOBRE LA DINAMICA DE CORTO
PLAZO DE PRODUCCION Y PRECIOS

El análisis macroeconómico convencional ha estado generalmente orientado a la determinación de magnitudes agregadas tales como producción, empleo, precios y tasas de interés. Dentro de dicho análisis, las recomendaciones de política monetaria y fiscal muchas veces se presentan contrapuestas dependiendo de si el supuesto que se hace es que el nivel general de precios es una variable exógena o si el producto real es exógeno. Suponer que una variable es exógena significa que a dicha variable se la considera independiente del resto de las variables que integran el modelo. De aquí, que el supuesto de que los precios o el producto sean exógenos resulta sumamente cuestionable.

La contribución de este trabajo radica en la incorporación de una relación, que en nuestro estudio llamamos oferta agregada, que permite el análisis del comportamiento del producto y niveles o tasas de cambio en precios en forma simultánea en el corto plazo. Específicamente, lo que esta relación propone es que los oferentes varían las cantidades ofrecidas en respuesta a cambios en el nivel general de precios que no fueron propiamente anticipados y erróneamente interpretados como un cambio en el precio real de su producto ^{1/}.

El párrafo anterior revela como aspecto importante el proceso por medio del cual se forman las anticipaciones de precios.

^{1/} Esta relación, que no es nueva en la literatura económica, está vinculada a los desarrollos más recientes del fenómeno conocido como "curva de Phillips". Originariamente la curva de Phillips se formuló en base a una regularidad empírica observada en Gran Bretaña. En base a esto se pensó que existiría una relación permanente entre inflación y desempleo por medio de la cual a una mayor tasa de inflación correspondería una menor tasa de desempleo y viceversa. Los fundamentos teóricos de este fenómeno son escasos y evidencia empírica más reciente indica que ni siquiera en Gran Bretaña es posible encontrar hoy en día un intercambio favorable entre inflación y producto o empleo como el observado por Phillips. La "oferta agregada" que discutimos en el texto de este trabajo corresponde a una reformulación más reciente de la curva de Phillips donde el ajuste de expectativas puede producir un intercambio favorable entre producción y empleo en el corto plazo.

Nuestro modelo adopta la hipótesis que las anticipaciones se forman tomando en cuenta toda la información disponible y como si el público conociera la teoría económica relevante que determina el nivel general de precios. Esto no necesariamente tiene que ser así: gran parte del público puede desconocer la teoría económica; la hipótesis simplemente significa que las expectativas agregadas del público en general son las mismas que las predicciones de la teoría económica relevante.

El siguiente ejemplo muestra el tipo de razonamiento que respalda nuestra hipótesis. Considérese el caso de un individuo que observa que el nivel general de precios está relacionado, entre otras cosas, con el comportamiento de la masa monetaria de la economía y que el público, en general, forma sus anticipaciones sin tener en cuenta este fenómeno (por ejemplo, supóngase al público formando sus anticipaciones en base a la trayectoria que los precios siguieron en el pasado). Es muy probable que la omisión por parte del público de la cantidad de dinero como variable importante para predecir precios introduzca errores sistemáticos en sus predicciones. Nuestro individuo, ajeno al público en general, podría aprovecharse de esta situación obteniendo un lucro a través de especulaciones con inventarios. O, alternativa-mente, este individuo podría vender al público ignorante de nuestro ejemplo sus servicios como buen predictor de precios. Es precisamente, la acción de este individuo lo que hace que en el agregado las expectativas del público no difieran de las predicciones de la teoría económica relevante.

El modelo teórico presentado en este trabajo explica en forma aceptable las series trimestrales de inflación y producto de Argentina y Brasil. Los resultados empíricos indican que en países como Argentina y Brasil, que se caracterizan por tener tasas de inflación altas y erráticas, ni siquiera en el corto plazo es posible observar una relación estable entre producción y precios. Este resultado debe constituir un llamado de atención a los encargados de política económica que pretenden manipular la demanda agregada de la economía con el propósito

/de obtener

de obtener una mejora temporaria en el nivel de empleo. Nuestro trabajo sugiere que dicho tipo de políticas pueden conducir a resultados desagradables puesto que aún en el corto plazo políticas expansivas pueden conducir a una aceleración de la inflación sin mejoras apreciables en la tasa de desempleo. Este resultado surge del hecho de que la mejora temporaria en el empleo ante estímulos en la demanda agregada se debe a la incorrecta anticipación de los oferentes que mal interpretan un cambio en el nivel general de precios como un cambio en sus precios relativos. Consecuentemente, mientras más errático sea el comportamiento de la demanda agregada mayor será la cautela de los oferentes en responder a cambios de demanda, diluyendo de esta manera el intercambio favorable entre producción y precios.

La última parte de este trabajo hace uso de los parámetros estimados del modelo para simular políticas alternativas de estabilización que ilustran la dinámica de corto plazo de producción y precios. El supuesto implícito en estas simulaciones es que existe un período durante el cual el público continúa formando sus anticipaciones en base a un proceso histórico de creación de dinero mientras que el gobierno cambia dicho proceso por uno nuevo. Las simulaciones muestran varios aspectos de los comúnmente observados en otros estudios empíricos. En particular, se observan períodos donde tanto la tasa de inflación como el desempleo aumentan. Este fenómeno conocido como "stagflation" puede ocurrir en respuesta a un esfuerzo de las autoridades monetarias para controlar la inflación. Es precisamente este fenómeno lo que hace decir a algunos críticos que el viejo remedio para curar la inflación (es decir el control de la emisión monetaria) no funciona.

Las simulaciones también ilustran la ventaja del "gradualismo" para estabilizar la tasa de inflación. Se muestra que una disminución abrupta en la tasa de crecimiento de la oferta de dinero produce una caída significativa en el producto, mientras que, una disminución gradual en la tasa de crecimiento de la oferta de dinero, tiene un menor impacto inicial sobre el producto.

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AN EMPIRICAL INQUIRY ON THE SHORT RUN DYNAMICS OF OUTPUT AND PRICES

1. Introduction

The purpose of this paper is to study the short run relationship between output and inflation in the context of a macroeconomic model. Although a considerable number of economists have studied this subject, mainly from the point of view of the Phillips' curve theory, most of them have made use of an ad-hoc hypothesis regarding the process through which expectations are formed. Other economists (e.g., Lucas, and Sargent and Wallace) have studied the same subject and have postulated a rational expectation hypothesis for analyzing the short run trade-off between inflation and output and for testing the "natural rate" hypothesis.

The analysis performed in this paper is similar to that of Lucas and Sargent and Wallace. In fact the analysis in Section 2 starts with the assumption that the model previously postulated by Sargent and Wallace (1975) is an appropriate theoretical framework for analyzing the short run relationship between prices and output. In that section the model to be used is presented as well as some of its main limitations and implications.

In Section 3 a summary of the results of the structural analysis of the model is presented as well as the estimation procedure followed in order to obtain the estimates for the structural equations of the system. These estimates, based on the available information for Argentina and Brazil, are presented in Section 4.

In Section 5 of this paper an attempt is made in order to analyze the short run dynamics of price and output based on the empirical findings of part 4. This obviously implies that the parameters of the model have to be assumed constant over the period of analysis. As shown below, the duration of this period is of particular importance given the assumption of rational expectations that the model has built in.

/2. The Macroeconomic

2. The Macroeconometric Model

As mentioned above, the model to be analyzed in this section is a standard macroeconomic model in which expectations will be assumed to be "rational" in the sense of Muth (1961). This assumption was incorporated in similar models by Sargent (1973) and Sargent and Wallace (1975). In this paper some modifications are introduced in order to arrive at a direct estimable relationship for a short run output-inflation trade-off.

The model consists of the following three equations:

(a) aggregate supply

$$y_t = y_{n,t} + a(p_t - {}_t p_{t-1}^*) + k(y_{t-1} - y_{n,t-1}) + u_{1t} \quad a > 0 \quad (1)$$

(b) aggregate demand

$$y_t = y_{n,t} + g + c(r_t - ({}_{t+1} p_{t-1}^* - {}_t p_{t-1}^*)) + u_{2t} \quad c < 0, \quad g > 0 \quad (2)$$

(c) portfolio balance

$$p_t = \phi m_t + y_t + b r_t + u_{3t} \quad -\infty < b < 0 \quad (3)$$

In these equations y_t , p_t and m_t are the natural logarithms of real income, the price level and the nominal stock of money, g is a constant and the $u_{i,t}$'s $i = 1, 2, 3$ are disturbance terms. The variable $y_{n,t}$ is a measure of normal productive capacity that will be represented by the trend in real output in the empirical application of the model. Therefore, $y_t - y_{n,t} = y_{c,t}$ represents cyclical or "detrended" output. The variable ${}_{t+1} p_{t-1}^*$ represents the public's expectation, at time t , of the logarithm of the price level expected to prevail at $t + 1$. The variable r_t is the nominal rate of interest.

Equation (1) is an aggregate supply equation relating detrended output to the gap between current price level and the public's prior expectation of the current price level. In this equation lagged detrended output indicates that deviations of aggregate supply from normal capacity may display some persistence. This same equation was postulated and used by Lucas (1973).

/Equation (2)

Equation (2) is an aggregate demand equation which relates the deviation of aggregate demand to the real rate of interest which in turn is represented by the nominal rate of interest minus the expected rate of inflation. This equation, used by Sargent and Wallace (1975), differs from the one used by Sargent (1973) in the definition of the real rate of interest. Sargent (1973) uses the usual definition, that is, $r_t - ({}_{t+1}p_t^* - p_t)$. Although no explanation can be found in that paper for the reasons for using the definition stated in equation (2), it seems plausible that demanders at time t do not observe p_t so they have to anticipate it. Notice that this is not inconsistent with equation (1) in the sense that suppliers do observe p_t because equation (1) does not imply that but it is derived from individual suppliers who only observe the prices in their markets and not the general price level (see Lucas (1973) pp. 327-328).

Some limitations of equation (2) are stated in Sargent (1973) as follow:

"an important thing about equation (2) is that it excludes as arguments both the money supply and the price level, ... This amounts to ruling out direct real balance effects on aggregate demand. It also amounts to ignoring the expected rate of real capital gains on cash holdings as a component of the disposable terms that belong in the expenditure schedules that underlie equation (2). Ignoring these things is usual in macroeconomic work."

Another aspect of this model is the lack of symmetry between equation (1) and (2), that is, only suppliers have explicit misperceptions of prices and only demanders have an explicit responses to changes in the real rate of interest. Implicitly, the effect of the neglected variables in each equation could be captured if they induced some stable stochastic process in the error terms.

Equation (3) is a demand for money relationship with unit real income elasticity (this assumption is not crucial and will be relaxed later on) that summarizes the condition for portfolio equilibrium. In other words, when equation (3) is satisfied owners

/of bonds

of bonds and equities are satisfied with the division of their portfolio between money (assumed to be exogenous), on the one hand, and bonds and equities on the other hand. ϕ is a polynomial in the lag operator (that is, $\phi = \phi_0 + \phi_1 L + \phi_2 L^2 + \dots$, where $\phi_0 + \phi_1 + \dots = 1$) introduced in an effort to capture the effects of lagged changes of m_t on nominal income ^{1/}. The degree of this polynomial will be determined empirically.

The u_{it} 's, $i = 1, 2, 3$ are random disturbances with zero means that may be serially and contemporaneously correlated.

On pure theoretical grounds there is not a strong justification for the existence of a lagged response of nominal income to changes in the quantity of money. However the existence of lags is confirmed in most of the empirical works that relate money and prices.

The working of the model can easily be illustrated leaving aside the problem of how expectations are formed. Consider for the moment only equations (2) and (3) that resemble the simple textbook model IS - LM. Equation (2) correspond to equilibrium in the real sector and relates real income to the real rate of interest, that is, the IS curve. Equation (3) refers to equilibrium in the monetary sector usually represented by the LM curve. The system formed by equation (2) and (3) is not determined because we have two equations and three endogenous variables, y , p and r_t . This problem is solved in the standard textbook analysis assuming either that prices are rigid so the shifting of the IS and LM curves affects

^{1/} Equation (3) can be derived from the simple quantity theory, that is

$$Y_t \cdot \frac{1}{V_t} = M_t \quad (3')$$

Now in order to capture the lagged effect of M_t on the left hand side we have to specify something like

$$Y_t \cdot \frac{1}{V_t} = f(M_t, M_{t-1}, M_{t-2}, \dots)$$

and an specific construction is

$$Y_t \cdot \frac{1}{V_t} = \exp(\phi L \ln M_t) \quad (3'')$$

where ϕ is a polynomial in the lag operator (notice that if $\phi = 1$ we get (3')).

Assuming $(1/V_t) = \exp(br_t)$ and taking logs on both sides of (3'') we get equation (3) of the text.

/real output

real output (this would be the simplest Keynesian model) or, assuming that the economy is in full employment so the shifting of the IS and LM curves would only affect prices (this would be the simplest quantity theory approach). In our case we solve the problem of joint determination of prices and output through equation (1) that in turn add one important feature to the model which is the relationship between output and price misperceptions.

To complete the model we should specify how expectations are formed. This is a delicate matter. It has been a custom in the economic profession to postulate different ad-hoc hypothesis about how expectations are formed. The most popular is Cagan Hypothesis of adaptive expectations, although the explanation for its use were confined to the fact that adaptive expectations seemed reasonable and proved useful in explaining data. The hypothesis of rational expectations used in this paper, that follows Muth (1961) proposal, consider that expectations are informed predictions of future events based on the available information and the relevant economic theory. This has a strong implication. With this assumption the economist that is modelling an economy does not have a superior knowledge of the "reality". This in turn is confirmed by the fact that actual expectations "are more accurate than naive models and as accurate as elaborate equation systems" (see Muth (1961), p. 316). Thus, our model is completed with the following equations.

$${}^t p_{t-1}^* = E p_t \quad (4)$$

$${}^{t+1} p_{t-1}^* = E p_{t+1} \quad (5)$$

where $E p_t$ is the conditional mathematical expectation of p_t formed using the model and all the information assumed to be available as of the end of period $t - 1$ (hereafter the E operator will always be conditional on the information available as of the end of period $t - 1$).

/After some

After some algebraic manipulations of the model (see Appendix A) two equations for expected prices can be obtained; these are

$$E p_t = \left[\frac{1}{1-b} \right] \sum_{j=0}^{\infty} \left[\frac{1}{1-b^{-1}} \right]^j \left[E \rho_{m,t+j} - y_{n,t+j} \right] + \left[\frac{J_3}{1-J_0} \right] \sum_{j=0}^{\infty} \left[\frac{1}{1-b^{-1}} \right]^j y_{c,t-1} + c_0 \quad (6)$$

and

$$E p_{t+1} = \left[\frac{1}{1-b} \right] \sum_{j=0}^{\infty} \left[\frac{1}{1-b^{-1}} \right]^{j+1} \left[E \rho_{m,t+j+1} - y_{n,t+j+1} \right] + \left[\frac{J_3}{1-J_0} \right] \sum_{j=0}^{\infty} \left[\frac{1}{1-b^{-1}} \right]^{j+1} y_{c,t-1} + c_0 \quad (7)$$

In these equations J_0 , J_3 and c_0 are constants that are complicated functions of the structural parameters of the model.

From these equations, it is easy to illustrate the process of formation of expectations; let us assume for a moment that $b = 0$ (that is, that the interest elasticity of the demand for money is zero). Then, after taking first differences (D operator), equation (6) can be reduced to

$$ED p_t = \beta_0 ED m_t - (\beta + k D y_{c,t-1}) + \beta_1 D m_{t-1} + \beta_2 D m_{t-2} + \dots \quad (8)$$

where β is the slope coefficient of the trend in real output and $J_3/(1-J_0) = -k$ when $b = 0$. Equation (8) clearly shows that the expected rate of change of prices depends upon the expected rate of change in the money supply in period t , the natural rate of growth in output (β), a term in the cyclical component of output in $t-1$, and past rates of change of the money supply. If $b \neq 0$ the results are not far from the quantity theory in expectation form although the algebraic expression representing the expectation formation process is more complicated. The money supply on the

basis of which the public makes its forecasts of the future path of m_{t+j} is of particular relevance ^{1/}.

1/ In searching for a process determining the money supply we can choose either to postulate a model for the money supply by relating it to a set of "predetermined variables" relative to the model (1) - (3) (so m_t still remains as if it were exogenous or determined outside of the system (1) - (3)) or, we can identify a Box-Jenkins ARIMA process. It has been customary in the economics profession to call these models "naive models" because of their rather simple structure by which only past values of a variable are used to predict future values of the same variable. However, it has recently been shown (see Zellner and Palm (1974)) that these models might not be naive at all. Indeed these models (the ARIMA models) represent the "final form" for a variable implied by a highly sophisticated model. I will briefly illustrate this point with a model for the nominal money supply. Let us assume that in a given country the money supply is generated by the following relationship

$$Dm_t = c_1 + a_1 Dm_{t-1} + b_1 Dg_t + e_1 Dx_t + v_t \quad (9)$$

where g_t could be the federal budget relative to lagged GNP, x_t could be the lagged balance of payment surplus relative to GNP and v_t an error term stochastically independent of the errors in the structural equations. In our case a_1 , b_1 and e_1 are assumed to be constants for simplicity, but in a more general analysis we could assume a_1 , b_1 and e_1 to be polynomials in the lag operator.

Now we shall show that equation (9) implies a final equation for m_t that is in the form of an ARIMA process. Equation (9) can be written as

$$(1 - a_1 L) Dm_t = c_1 + b_1 Dg_t + e_1 Dx_t + v_t \quad (10)$$

Now the predetermined variables g_t and x_t can follow any process over time, that is, both could follow a random walk or one could follow a random walk and the other a given ARIMA process etc. To illustrate the problem at hand we will assume that

$$Dg_t: \text{ARIMA } (2, 1) \text{ or } \phi(2) Dg_t = \theta(1) v_{1t}$$

$$Dx_t: \text{random walk or } \phi(0) Dx_t = \theta(0) v_{2t}$$

Where the v 's are stochastically independent of the disturbances in the structural equations. Multiplying both sides of (10) by $\phi(2)\phi(0)$ we have

$$(1 - a_1 L)\phi(2)\phi(0) Dm_t = \phi(2)\phi(0)c + b\phi(0)\theta(1)v_{1t} + e\phi(2)\theta(0)v_{2t} + \phi(2)\phi(0)v_t \quad (11)$$

(cont.)

/The empirical

The empirical analysis of section 4 considers two processes as determining the money supply; an ARIMA process that in its "inverted form" is

$$Dm_t = \tilde{\pi}_1 Dm_{t-1} + \tilde{\pi}_2 Dm_{t-2} + \tilde{\pi}_3 Dm_{t-3} + \dots + v_{3t} \quad (12)$$

where the $\tilde{\pi}$'s are parameters. The second process will be a model of the form

$$Dm_t = \tilde{\pi}'_1 Dm_{t-1} + \tilde{\pi}'_2 z_t + u_t \quad (13)$$

where $\tilde{\pi}'_1$ can be a parameter or a polynomial in the lag operator and $\tilde{\pi}'_2$ can be a row vector of parameters or a row vector of polynomials in the lag operator while z_t is a column vector of predetermined variables.

The empirical tests will not be carried out directly in the form of equations (12) and (13) but indirectly through the transfer functions of the next section.

1/ (cont.) In this last expression we notice that we have obtained an ARIMA (3, 2) process (if no cancellation occurs) for m_t as implied by equation (9) and the assumption for the predetermined variables g_t and x_t . This clearly illustrates that if we obtain the process ARIMA (3, 2) for m_t this is not a naive model at all, but on the contrary it could be reflecting the "true" model governing the behavior of the money supply.

Now we go back to our original problem of finding a process for m_t on the basis of which the public makes its forecasts of the future path of the money supply. The above discussion demonstrates that we cannot talk about "alternative models" when we evaluate a model of the sort of equation (9) with respect to a model like (11) because this could be the final form of (9). Nevertheless, we have considered it appropriate to check empirically the ARIMA hypothesis for m_t as well as a model of the sort implied by equation (9), however no further attention is dedicated to the "theory of the money supply" that underlies our hypothesis of the money supply process, a subject that goes beyond the scope of this paper.

3. Towards an Empirical Test of the Model

In this section we outline the method followed in order to get the estimates for the structural equations of the model. Thus two points are jointly developed, one is the computation of expected prices and the other is the endogeneity of p_t that precludes the straight forward estimation of equation (1) using ordinary least squares.

Some testable implications of the model can be derived from a structural analysis of the system. This analysis, following the method suggested in Zellner and Palm (1974) is presented in Fernandez (1975) where the final equations of the system (1) - (5) were derived and checked with the data. Also in that work a variant of the system is analyzed in which an adaptive expectation hypothesis was used for prices. This version was incompatible with the available information for both Argentina and Brazil while the rational expectations version of the model (system (1) - (5)) was compatible under certain conditions ^{1/}.

At the estimation stage we shall concentrate on equations (1) and (3). The main problem with equation (2) is the variable r_t for which we do not have data for some countries (for example Argentina and Brazil). This problem is eliminated in equation (3) because it

^{1/} As it is shown in Fernandez (1975), given a system of structural equations we can work out the "final equations" for the variables of the system. These are in the form of ARIMA processes. On the other hand, we can identify the ARIMA processes for the variables using the available information on each variable. If the structural equations of the model are correct, the final equations derived for each endogenous variable should have the same structure that the ARIMA processes identified for those variables from the available information. If this is the case, we say that the model is compatible with the available information

is assumed that variation in r_t is dominated by variation of the expected rate of inflation and the public's forecast of the rate of inflation (based on information available at $t - 1$) is used as a proxy for r_t . It is obvious that this substitution cannot be made in equation (2), the term $(r_t - ({}_{t+1}p_{t-1}^* - {}_t p_{t-1}^*))$ would vanish when r_t is substituted by ${}_{t+1}p_{t-1}^* - {}_t p_{t-1}^*$. Nevertheless, the system formed by equations (1) and (3) is perfectly determined when a proxy is used for r_t , let us say, $r_t' = Dp_t^* = {}_{t+1}p_{t-1}^* - {}_t p_{t-1}^*$. For convenience, we write equations (1) and (3) again:

$$y_{c,t} = a(p_t - {}_t p_{t-1}^*) + ky_{c,t-1} + u_{1t} \quad a > 0 \quad (1)$$

$$p_t + y_t = \phi m_t - br_t' - u_{3t} \quad -\infty < b < 0 \quad (3)$$

where $y_{c,t} = y_t - y_{n,t}$ (detrended output).

Let us consider first equation (1). We know that a direct estimation of this equation is not possible because p_t and y_t are jointly determined and ${}_t p_{t-1}^*$ is not observable. Thus, in this section our objective is to obtain an estimable relationship in place of (1), making use of the relationships previously developed.

In equation (6) we obtained an expression for the expectations formation process in which the expected log of the price level in period t was determined by the log of the money supply expected to prevail in period t , by the trend in the log of real output and by the detrended log in real output in period $t-1$. Clearly, the actual log of the price level differs from the expected value by a random component, let say, u_{4t} , so we can write

$$p_t = E p_t + u_{4t} \quad (14)$$

Then, our hypothesis implies that the expected p_t is computed as if the public attempted to obtain an optimal unbiased forecast of p_t using equation (6). Combining (14) and (6) we can write

$$/p_t =$$

$$p_t = \left[\frac{1}{(1-b)} \right] \sum_{j=0}^{\infty} \left[\frac{1}{(1-b^{-1})} \right]^j \left[E\phi_{t+j} - y_{n,t+j} \right] + \\ + \left[\frac{J_3}{(1-J_0)} \right] \sum_{j=0}^{\infty} \left[\frac{\alpha}{(1-b^{-1})} \right]^j y_{c,t-1} + c_0 + u_{4t} \quad (14a)$$

In (14a) we have a term in $E\phi_{t+j}$. Developing this term for $j = 0, 1, \dots$ taking expectations and recalling that $\phi = \phi_0 + \phi_1 L + \phi_2 L^2 + \dots$, we have

$$E\phi_{t+j} = E\phi_0^m t + \phi_1^m t-1 + \phi_2^m t-2 + \dots \quad j = 0$$

$$E\phi_{t+j} = E\phi_0^m t+1 + E\phi_1^m t + \phi_2^m t-1 + \dots \quad j = 1$$

$$E\phi_{t+j} = E\phi_0^m t+2 + E\phi_1^m t+1 + E\phi_2^m t + \phi_3^m t-1 + \dots \quad j = 2$$

Recall that the E operator is conditional on the information in period $t-1$, so $E m_{t-1} = m_{t-1}$ and so on for periods before period $t-1$. Now provided that we use the process (12) to obtain $E m_{t+j}$, $j = 1, 2, \dots$, we notice that the forecasts of m_t are obtained through linear combinations of $m_{t-1}, m_{t-2}, m_{t-3}, \dots$. These linear combinations should be combined with the other terms in $m_{t-1}, m_{t-2}, m_{t-3}, \dots$ that appear because of the lagged response of prices to changes in m_t , and with $y_{n,t+j}$, $j = 1, 2, \dots$ and $y_{c,t-1}$ to forecast p_t . Then we can rearrange the terms in $m_{t-1}, m_{t-2}, m_{t-3}, \dots$, and rewrite (14a) as

$$p_t = v(L)Lm_t - (1/(1-b)) \sum_{j=0}^{\infty} (1/(1-b^{-1}))^j y_{n,t+j} + \\ + (J_3/(1-J_0)) \sum_{j=0}^{\infty} (\alpha/(1-b^{-1}))^{j+1} + u_{4t} \quad (14b)$$

where $v(L)$ is a polynomial in the lag operator ($Lx_t = x_{t-1}$) capturing the effect of all the linear combinations on past values of m_t on p_t .

The first difference form of this equation is

$$Dp_t = v(L)LDm_t + h_0 Dy_{c,t-1} + c + u_{5t} \quad (14c)$$

where c accounts for the term in $y_{n,t+j}$ after differencing (recall that $y_{n,t}$ is a trend and differencing it yields the slope coefficient of the trend line) and h_0 represents the coefficient of $y_{c,t-1}$.

To estimate (14c), we have to consider the problem of collinearity, especially in the case of quarterly data, where a reasonable lag of two years would imply that m_t should be lagged eight times. A way of dealing with equation (14c) is to consider it to be a multiple input transfer function ^{1/}. The transfer function form of equation (14c) can be parsimoniously (in terms of the number of parameters) represented by

$$Dp_t = \frac{w_1(L)}{\alpha_1(L)} LDm_t + \frac{w_2(L)}{\alpha_2(L)} LDy_{c,t} + \frac{\theta(L)}{\beta(L)} u_t + c \quad (15)$$

The estimation of (15) can be done using the Marquardt algorithm, and the forecast made using the estimated version of (15) are minimum mean square error forecasts. Then, from (15) we can obtain a series of "expected prices"; we need now to compute a series of "actual prices". Let me recall that in equation (1) we cannot compute the difference $p_t - {}_t p_{t-1}^*$ and estimate that relationship because in our model both, p_t and $y_{c,t}$ are endogenous. Also by straightforward algebra (using (6), (7) and (12)) we obtain

$$Dp_t = \frac{w'_1(L)}{\alpha'_1(L)} Dm_t + \frac{w'_2(L)}{\alpha'_2(L)} LDy_{c,t} + \frac{\theta'(L)}{\beta'(L)} u'_t + c' \quad (16)$$

where the meaning of the notation is the same as in equation (15). Notice that the main difference between (15) and (16) is that in (15) m_t appears lagged one period.

^{1/} The analysis of transfer functions can be found in Chapters 10 and 11 of Box and Jenkins (1970). A derivation of the transfer function different from the one presented in this paper, for a simultaneous equation model can be found in Zellner and Palm (1974).

Now let me recall that equation (12) represents the hypothesis that the money supply follows an ARIMA process. If we use the assumption (13) for the money supply then equation (15) and (16) should be extended to include terms in the components of z_t .

Although the algebraic analysis is rather long, its intuitive interpretation is quite simple and straightforward. The rational expectation feature of the model implied that the public in forming their expectations uses the information available as of the end of period $t - 1$. In forming these expectations the money supply expected to prevail in future periods is important and it is assumed that the public forecasts future values of m_t by considering the history of m_t available at $t - 1$ (as well as other variables if (13) is used). But the history of m_t is not only relevant for forecasting future values; the recent past values of m_t also directly affect the price level because of the lagged response of prices to changes in the money supply. This is also considered in the expectations formation process. Equation (15) is oriented to capture this process.

Equation (16), although very similar to (15), is quite different. It is a reduced form for p_t implied by the system (1) - (5) and the assumption in (12) or (13) for the money supply. In (16), m_t directly affects the price level. The economy as a whole need not forecast m_t ; it is an exogenous variable determined by monetary authorities in period t which will have an immediate effect on p_t .

The fitted values for p_t from (16) will be introduced in (1) in place of p_t and the fitted values of (15) will be introduced in (1) in place of ${}_t p_{t-1}^*$ in order to estimate equation (1).

Consider now equation (3). This equation assumes that the real income elasticity of demand for money is one. This assumption need not be maintained since all the algebraic expressions that we obtained before can be rearranged to include an additional parameter (the real income elasticity of the demand for money). Hereafter we will relax this assumption writing (3) as

$$/Y_t =$$

$$Y_t = \phi m_t - br_t' - u_{3t}$$

where $Y_t = p_t + iy_t$, i representing the real income elasticity of demand for money. It should be noticed that if $i = 1$ then Y_t is the log of nominal income. Then the system (1) - (3) can be interpreted as follows: equation (3) determines nominal income and equation (1) determines the division of nominal income between changes in prices and changes in output.

For analyzing the cases in which $i \neq 1$ we will evaluate the results for three cases: $i = 0.5, 1.5$ and 2 . An attempt was made to estimate i using as instrumental variable for Dy_t and ARIMA process; however the results were not reliable because y_t behaves almost like a random walk. At the estimation stage, equation (3) will be expressed in the form of a transfer function with all variables in first difference. The transfer function form will allow us to estimate the lag structure induced by the polynomial lag operator ϕ .

4. Empirical Results

In this section we proceed to test and estimate the model presented in section 2 and 3 with the available data for Argentina and Brazil. Firstly we construct a series of expected prices on basis of the results obtained in fitting equation (15). Secondly, we construct a series of actual prices from the reduced form for prices, that is, equation (16) (recall that this step is necessary in order to avoid the problem of simultaneity in estimating equation (1)). "Actual prices" minus expected prices give us the misperceptions of prices that is needed to estimate equation (1). Finally, we estimate equation (3) under different assumptions with respect to the real income elasticity of the demand for money and using a proxy for the interest rate.

4.1 The Data

All the data for Argentina were obtained from International Financial Statistics (International Monetary Fund). They include quarterly data for the index of industrial production, wholesale prices, currency and demand deposits, wages set in collective bargaining and the balance of trade (all seasonally adjusted by the method of moving averages). The observations relate to the period 1956-I to 1973-II (this period was chosen in order to base the analysis on the maximum number of observations available for the index of industrial production) ^{1/}.

The log of the index of industrial production for Argentina was detrended splitting the data into two parts: from 1956-I to 1962-IV and from 1963-I to 1973-II. This was done because in the first period apparently there is no trend in real output and if a

^{1/} The index of industrial production is used for Argentina as a proxy for real income because is more reliable and complete than existing series of real output. For Brazil the only available information corresponds to real output.

single trend line were fitted to the whole period we would lose most of the cyclical fluctuations ^{1/}.

The data for Brazil were obtained from two sources: International Financial Statistics (IMF) and Goncalves (1974). From International Financial Statistics we obtained the series of wholesale prices (that excludes coffee) and currency and demand deposits. From Goncalves (1974) we obtained a series of real output. All the observations relate to the period 1955-I to 1971-IV (this period was chosen in order to base the analysis on the maximum number of observations available for real output). The data were seasonally adjusted by the method of moving averages.

All the variables were expressed in first difference of logs prior to estimation except in the case of balance of trade. This variable was computed as the log of exports minus the log of imports (this because of the impossibility of taking log of a negative number in the case of trade deficits).

The estimation of transfer functions was carried out using Marquardt's (1963) algorithm.

4.2 Estimates of the Transfer Function for Expected Prices

In Table 1 we present the estimates obtained for equation (15), which is the expression that determines expected prices ^{2/}. These

^{1/} As a matter of fact this was exactly the procedure originally followed. The procedure was abandoned because the detrended output obtained in this manner showed an initial period in which output was mostly above the trend, a second period of almost "seven years" in which output was below the trend and a third period where output was above the trend. A detailed explanation about some institutional aspects that could explain the difference in the trend of real output above mentioned can be found in Fernandez (1975), pp. 36-39.

^{2/} The column headed "dummy" correspond to the constant c in equation (15). The dummy appears in the empirical results for Argentina because the constant c is a term in the slope coefficient of the trend line for output. As we splitted the data in two periods and in each period there is a different slope coefficient a dummy with value of one from 1956-I to 1962-IV and two from 1963-I to 1973-II incorporated in the transfer function in order to capture the effect of the change in trend.

Table 1
ESTIMATED TRANSFER FUNCTIONS FOR EXPECTED PRICES

Model	Residual Sum of Squares (RSS)	Degree of Freedom (DF)	RSS/DF	Estimates of the AR and MA Parts of Dm_{t-1}	Estimates of $Dy_{t,t-1}$	Dummy or constant	Wages	B. of Trade	Estimates of the AR and MA Parts of u_t	Adj. R ²	
				<u>Argentina 1956-I-1973-II</u>							
(1)	0.122293	59	0.00207	0.492 (0.129) $\frac{1 - 1.041L + 0.785L^2}{(0.082)} (0.087)$	-0.006 (0.094)	0.011 (0.012)			$\frac{1}{1 - 0.402L - 0.252L^2}$ (0.133) (0.130)	.47	
(2)	0.117639	57	0.00206	0.693 - 0.317L (0.166) (0.221) $\frac{1 - 1.112L + 0.719L^2}{(0.135)} (0.102)$	-0.060 (0.104)	0.011 (0.013)			$\frac{1}{1 - 0.376L - 0.271L^2}$ (0.135) (0.131)	.48	
(3)	0.116154	59	0.00197	0.613 (0.146) $\frac{1 - 1.009L + 0.741L^2}{(0.099)} (0.098)$	-0.022 (0.091)	0.006 (0.009)	0.044 (0.081)	-0.004 (0.001)	$\frac{1}{1 - 0.436L}$ (0.124)	.49	
				<u>Brazil 1955-I-1971-IV</u>							
(1)	0.066609	60	0.00111	0.188 (0.122) $\frac{1 - 0.723L}{(0.174)}$	-0.009 (0.077)	0.016 (0.024)			$\frac{1}{1 - 0.520L}$ (0.116)	.46	
(2)	0.065183	59	0.00110	0.119 (0.099) $\frac{1 - 1.388L + 0.556L^2}{(0.662)} (0.578)$	-0.012 (0.078)	0.013 (0.023)			$\frac{1}{1 - 0.508L}$ (0.116)	.46	

Note: This table presents the estimates of equation (15). The terms AR and MA represent the autoregressive and moving average parts respectively of the rational polynomials. Figure in parentheses are large sample standard errors.

models have been selected from a larger number of models with different lag structures and different error terms. The selection has been carried out using the likelihood ratio test proposed by Zellner and Palm (1974) (see Fernandez (1975) for a description and application of this test).

Models (1) and (2) for Argentina assume that the money supply follows a process as represented by equation (12) and model (3) considers the assumption implied by (13). In model (3) we have computed the transfer function with wages and balance of trade as input variables. We notice from Table 1 that in the case of Argentina there is a slight reduction in the RSS/DF and a small increase in the adjusted R^2 when passing from model (1) or (2) to model (3) ^{1/}. At the bottom of the table we present the results obtained for Brazil where an insignificant reduction in the RSS/DF is observed when we go from the simple lag structure of model (1), to the more complex lag structure of model (2).

4.3 Estimates of the Reduced Form for Prices

Table 2 shows the estimates of the transfer functions for prices (that is, equation (16)). Here again, for the case of Argentina model (1) and (2) incorporate assumption (12) for the money supply while model (3) incorporate assumption (13). The models (1) and (2) are the best results obtained for each hypothesis regarding the money supply respectively. In both, Table 1 and Table 2, the coefficient of the Balance of Trade variable is significantly different from zero at the 5 per cent level. Only in Table 2 does the Balance of Trade variable have a coefficient estimate with an algebraic sign that the theory predicts (that is positive sign).

^{1/} The adjusted R^2 reported in the tables for transfer functions takes account of the correction for degree of freedom. That is,
 $1 - R^2_{adj} = \frac{n - 1}{n - k} (1 - R^2)$.

Table 2
ESTIMATED TRANSFER FUNCTIONS FOR PRICES

Model	Residual Sum of Squares (RSS)	Degree of Freedom (DF)	RSS/DF	Estimates of the AR and MA Parts of Dm_t	Estimates of Dy_{t-1}	Dummy or constant	Wages	B. of Trade	Estimates of the AR and MA Parts of u_t	Adj. R^2
(1)	0.125758	59	0.00213	$\frac{0.422}{(0.112)}$ $\frac{1 - 1.216L + 0.773L^2}{(0.102) (0.103)}$	0.049 (0.095)	0.004 (0.012)			$\frac{1}{1 - 0.400L - 0.216L^2}$ (0.133) (0.132)	.45
(2)	0.116639	57	0.00205	$\frac{0.224 + 0.346L}{(0.160) (0.202)}$ $\frac{1 - 1.073L + 0.763L^2}{(0.104) (0.091)}$	0.090 (0.105)	0.004 (0.012)			$\frac{1}{1 - 0.365L - 0.280L^2}$ (0.135) (0.132)	.48
(3)	0.113562	57	0.00199	$\frac{0.207 + 0.435L}{(0.180) (0.229)}$ $\frac{1 - 1.054L + 0.749L^2}{(0.116) (0.098)}$	0.010 (0.095)	-0.003 (0.010)	0.018 (0.084)	0.003 (0.001)	$\frac{1}{1 - 0.438L}$ (0.124)	.49
(1)	0.064908	61	0.00106	$\frac{0.212}{(0.117)}$ $\frac{1 - 0.716L}{(0.155)}$	-0.004	0.010 (0.023)			$\frac{1}{1 - 0.511L}$ (0.113)	.47
(2)	0.064130	60	0.00107	$\frac{0.119}{(0.190)}$ $\frac{1 - 1.389L + 0.546L^2}{(0.630) (0.611)}$	-0.005	0.008 (0.022)			$\frac{1}{1 - 0.498L}$ (0.115)	.47

Note: This table presents the estimates of equation (16). The terms AR and MA represent the autoregressive and moving average parts respectively of the rational polynomials.

For the case of Brazil we observe again that no appreciable reduction in the RSS/DF is obtained in going from the simple lag structure of model (1) to the more complex lag structure of model (2).

In both, Table 1 and Table 2, the estimates for the variables $Dy_{c,t-1}$ and dummy or constant are small numbers not significantly different from zero. This is not in contrast with the theoretical model because it is shown in Appendix A that these parameters can indeed be close to zero (see equations (A.12) and (A.13) in Appendix A).

4.4 Estimates of the Aggregate Supply

Recall that Table 1 provides the estimates of equation (15) which in turn allow us to obtain a series of "expected prices" needed to estimate equation (1) of our original model. By the same token equation (16), whose estimates are given in Table 2, provides us with a series of "actual prices" to estimate equation (1). Then the next step is to compute a one step ahead forecast from (15) that would give us a proxy variable for ${}_t p^*_{t-1}$; similarly a one step ahead forecast from (16) would give us a proxy for p_t . The difference between the proxy of p_t and the proxy for ${}_t p^*_{t-1}$ is introduced in equation (1) in place of $(p_t - {}_t p^*_{t-1})$ and the estimation of this equation provides us with an estimate of the slope coefficient of our short run Phillips equation. Then Table 3 shows the results obtained by this procedure and indicates the different models used for forecasting prices and reduced forms used for prices. On testing the significance of the Phillips parameter for Argentina using a two-tailed test we notice that at the 5 per cent level only regression (5) shows an estimate significantly different from zero. Using a one tailed test (that is the alternative hypothesis is that the parameter is greater than zero), estimates of regressions (3), (4) and (5) provide evidence for rejecting the null hypothesis at the 5 per cent

/level of

level of significance. In all the cases the Box and Pierce Q statistic is in favor of rejecting the hypothesis of autocorrelation of residuals ^{1/}.

Perhaps it is convenient at this stage to take a closer look at the estimates of Table 3. Let me recall that the estimate of parameter "a" is an estimate of the slope of the Phillips curve. Our results for Argentina indicate that there is some evidence in favor of a short run trade-off between inflation and output given by the 95 per cent confidence intervals for the estimates of regressions (3), (4) and (5). These are (-0.089, 0.991), (0.0, 1,384) and (0.001, 2,268) respectively. Now the short run trade-off that we have found is not in contrast with the natural rate hypothesis of Friedman because, as equation (1) indicates, if prices are anticipated correctly output will remain in its long run trend (or "natural" level).

On the other hand, our results do not provide evidence either in favor of the naive Phillips curve approach or in favor of the Solow-Tobin analysis. The naive Phillips' curve approach says that there is only one Phillips' curve indicating a positive trade-off between inflation and output regardless of expectations. The Solow-Tobin analysis says that people adjust to changes in prices, but they are subject to some money illusion that allows for a permanent trade-off between inflation and output. Both of these hypotheses are ruled out in our set up that specifies that when actual prices are equal to expected prices output will remain at its long run natural level.

^{1/} The Q statistic is calculated from the first K autocorrelations \hat{r}_k ($k = 1, 2, \dots, K$). If the fitted model is appropriate,

$$Q(K) = n \sum_{k=1}^K \hat{r}_k^2$$

is approximately distributed as $X^2 (k - p - q)$. If the model is wrong the value of Q will be inflated. In the case of Table 3, $p = q = 0$ because there are not autoregressive or moving average parameters in the noise model.

Table 3
ESTIMATES OF THE AGGREGATE SUPPLY EQUATION

Model for Reduced Form	Model for Expected Prices	a	k	Adj R ²	Q Statistics
<u>Argentina 1956-I - 1973-II</u>					
(1) M ₂	M ₂	0.877 (0.594)	0.564 (0.102)	0.95	15.3
(2) M ₂	M ₁	0.647 (0.578)	0.574 (0.102)	0.94	16.5
(3) M ₁	M ₁	0.401 (0.295)	0.575 (0.102)	0.35	15.7
(4) M ₁	M ₂	0.692 (0.346)	0.569 (0.101)	0.36	14.2
(5) M ₃	M ₃	1.140 (0.564)	0.778 (0.102)	0.35	15.1
<u>Brazil 1955-I - 1971-IV</u>					
(1) M ₁	M ₁	-0.272 (0.919)	0.664 (0.095)	.430	15.1
(2) M ₁	M ₂	-0.492 (0.750)	0.660 (0.095)	.433	16.0
(3) M ₂	M ₁	0.842 (1.328)	0.657 (0.095)	.433	14.5
(4) M ₂	M ₂	-0.140 (1.390)	0.665 (0.095)	.429	14.9
(5) Actual Prices	M ₁	0.168 (0.178)	0.694 (0.099)	.437	15.8
(6) Actual Prices	M ₂	0.153 (0.172)	0.638 (0.099)	0.436	15.5

Note: Chi-Square values from table:

$$\chi^2(24) = 33.2 \quad 0.10 \text{ level of significance}$$

$$\chi^2(24) = 36.4 \quad 0.05 \text{ level of significance}$$

The models used to represent prices and expected prices are symbolized in this Table with the letter M and a subindex. Thus, M₂ in the column headed "Model for Reduced Form" means that model (1) of Table 2 is being used to represent actual prices in the aggregate supply equation.

In the case of Brazil we notice that in equations (1), (2) and (4) the estimate of "a" is negative although not significantly different from zero at the 0.05 level in a two-tailed test. Model (3) presents the right sign but its "a" estimate has a large standard error that makes it not significantly different from zero. In all the cases the value of the Q statistics favor rejection of the hypothesis of autocorrelation in the residuals.

In order to compare our results with other results obtained for Brazil by Goncalves (1974), we estimated the last two models of Table 3 where the actual prices were included instead of the forecast of the reduced form for prices. Goncalves, did a similar estimation under the assumption that the price level was exogenously determined (mainly due to strongly enforced price controls in most of his period of analysis). He worked with the period 1959-1969 and used another hypothesis for expectations formation. His results provide an estimate of α equal to 0.41 (standard errors are not reported in his work). Other of this results shows α equal to 0.27 when a dummy variable is included with a value of unity from 1961-I to 1963-II and zero elsewhere (this dummy variable is supposed to capture the effect of price controls). It should be noted that this last result, obtained by Goncalves, is close to our estimates as in model (5) and (6) of Table 3. From our results for Brazil we must conclude that the empirical evidence is not clearly in favor of a stable short run trade-off between output and inflation even in the short run ^{1/}.

^{1/} It is important to mention here one interesting result obtained in the work of Lucas (1973). He found, in a sample of 18 countries and working with annual observations, that "in a stable price country like the United States, policies which increases nominal income tend to have a large initial effect on real output, together with a small positive initial effect on the rate of inflation. Thus the apparent short-term trade-off is favorable, as long as it remains unused. In contrast, in a volatitl price country like Argentina, nominal income changes are associated with equal, contemporaneous price movements with no discernible effect on real output"(see Lucas (1973) pp. 332-333). Our results for Argentina and Brazil tend to confirm this finding and the underlying theory that specify that a favorable trade-off between output and inflation depends upon "fooling" suppliers, a thing that becomes hard to do when the variance of the demand shifts becomes large.

4.5 Estimates of the Transfer Function for Nominal Income

Now, we proceed to the estimation of equation (3) of our original model. Let me recall that in this equation we are using nominal income as the dependent variable when the real income elasticity of the demand for money is assumed equal to one, and we are using as dependent variable the term $p + iy$ where i is the real income elasticity, for all the cases in which it is assumed that $i \neq 1$.

Also, we are using the first difference in the one step ahead forecast for prices from model (1) of Table 1 as a proxy for the nominal rate of interest. The estimates for these transfer functions are presented in Table 4. In this table it is shown that in the case of Argentina when i is greater than one both the degrees of the polynomials estimated and the RSS/DF, are higher than when i is equal to or lower than one. I have no explanation for this except, as I mentioned above, Dy_t is a very noisily series and as i becomes large it magnifies the noise of the series of "nominal income". The last transfer function reported in Table 4 includes a second order autoregressive process for the error term which introduces an appreciable reduction in the RSS.

In the case of Brazil when the real income elasticity is relatively large (1 or 1.5) the adjusted R^2 's are low. The best explanation is obtained with $i = 0.5$ and with a second order polynomial in the disturbance term.

It should be noticed that if the estimate for the parameter "a" is assumed to be zero and if the real income elasticity of the demand for money is assumed to be one, our system is reduced to a special formulation of the Theory of Nominal Income of Friedman.

This can be explained as follows: if "a" is assumed equal to zero equation (1) cannot longer be used to break down the changes in nominal income obtained from equation (3) between changes in prices and output. So what our system explains is just nominal income.

Table 4
ESTIMATED TRANSFER FUNCTIONS FOR NOMINAL INCOME

Model	Residual sum of squares (RSS)	Degree of freedom (DF)	RSS/DF	Estimates of the AR and AM Parts of Dm_{t-1}	Estimates of the AR and MA Parts of Dr'_t	Estimates of the AR and MA Parts of u_t	Adj. R^2
<u>Argentina 1956 I - 1973 II</u>							
(1) (1 = 1.5)	0.249863	56	0.00446	$1.021 - 0.264L + 1.283L^2$ (0.269) (0.311) (0.247)	0.565 (0.311)		.27
				$1 + 1.739L + 0.596L^2$ (0.208) (0.234)			
(2) (1 = 1)	0.140220	58	0.00242	$0.406 - 0.147L$ (0.149) (0.182)	0.504 (0.213)		.38
				$1 - 1.496L + 0.737L^2$ (0.120) (0.106)			
(3) (1 = 0.5)	0.139771	58	0.00241	$0.424 - 0.130L$ (0.165) (0.212)	0.475 (0.215)		.38
				$1 - 1.496L + 0.737L^2$ (0.120) (0.106)			
(4) (1 = 0.5)	0.121832	54	0.00225	$0.442 - 0.141L$ (0.174) (0.260)	0.266 (0.216)	1 $1 - 0.276L - 0.171L^2$ (0.141) (0.150)	.41
				$1 - 1.407L + 0.722L^2$ (0.176) (0.141)			
<u>Brazil 1955 I - 1971 IV</u>							
(1) (1 = 1.5)	0.377770	57	0.00593	$0.212 + 0.103L + 0.348L^2$ (0.248) (0.526) (0.521)	-0.093 (0.516)		.093
				$1 - 0.383L$ (0.422)			
(2) (1 = 1)	0.182688	57	0.00320	$0.253 - 0.272L + 0.356L^2$ (0.255) (0.389) (0.359)	0.115 (0.380)		.171
				$1 - 0.419L$ (0.306)			
(3) (1 = 0.5)	0.095822	57	0.00168	$0.292 - 0.161L + 0.366L^2$ (0.184) (0.285) (0.243)	0.723 (0.375)		.293
				$1 - 0.466L$ (0.213)			
(4) (1 = 0.5)	0.086180	53	0.00162	$0.197 - 0.061L + 0.221L^2$ (0.182) (0.272) (0.251)	0.085	1 $1 - 0.223L - 0.169L^2$ (0.167) (0.169)	.342
				$1 - 0.613L$ (0.226)			

In this case, equation (15) determines the prices expectations (and still under the hypothesis of rational expectations) that would dominate the changes in the nominal rate of interest in equation (3). Let me recall that from (15) we obtain the proxy Dr'_t for the nominal rate of interest.

Section 5, which analyzes the short run dynamics of prices and output makes use of the estimates of this section. In choosing the estimates that will represent our model we have made use of models (1) of Table 1 and Table 2 for Argentina and models (1) of Table 1 and (2) of Table 2 for Brazil which are the models that most appropriately represent the process for expected prices and prices, respectively, under the hypothesis of equation (12) for the money supply. This implies that equation (3) of Table 3 (for both, Argentina and Brazil), was used for representing the Phillips' equation. Finally the model (2) ($i = 0.5$) (Argentina) and (1) ($i = 0.5$) (Brazil) of Table 4 were used as a transfer functions for nominal income.

5. The Short Run Dynamics of Output and Prices

In this final section we want to analyze the short run behavior of a system like that developed in Section 2. In doing so we will perform a deterministic simulation changing the rate of growth of the money supply in order to observe the short run behavior of the endogenous variables of the system.

Before going on along the lines of the previous paragraph we will sketch some propositions in the literature about some possible paths of this short run dynamics or adjustment process.

For this adjustment process we understand the path followed for the variables from one long run equilibrium position to other long run equilibrium position when an exogenous force shocks the system. In our simulations the shock will be a shift in the rule governing the money supply. These long run equilibrium positions have been already stated in the literature for models of this kind. So for example, if the real income elasticity of the demand for money is unity and if we change the rate of growth of the money supply from 3 per cent to 10 per cent then the long run equilibrium position of nominal income will shift from 3 per cent to 10 per cent (this is no more than the quantity theory; for a discussion of this proposition see Friedman (1971), pp. 56-58). Then the question to be answered by the analysis of the short run dynamic of the system is how the endogenous variables, for example, nominal income, moves from the 3 per cent position to the 10 per cent position.

Now, in order to show the short run dynamics of prices and output implied by this model we will use three different simulations concerning the behavior of the money supply. However, before proceeding, it is necessary to emphasize the circumstances under which these simulations can cast some light on the short run dynamics of prices and output. Our model, under the hypothesis implied by equation (12) about the money supply, states that all the parameters of equation (15) are stable as long as the process followed by the money supply is the same. That is, if the money supply has followed an ARIMA (3, 1, 2)

/process, then

process, then the forecasts will be accurate as long as this process stays the same, mainly because the people compute their expectations as if they knew the process ARIMA (3,1,2) governing the money supply. If we change this process then the parameters of equation (12) will eventually change and consequently the parameters of equation (15). This can be proved mathematically. However, there also is a clear intuitive explanation. We cannot expect that people will continue indefinitely to form their expectations on the basis of one process ARIMA (3,1,2) for the money supply, when the monetary authorities have changed the rule governing the money supply to, let us say, a rate of growth of m_t at a 10 per cent per period. If this second rule has been in operation for a long enough time period, then in computing their expectations people will use the process $Dm_t = 0.10$ instead of the previous ARIMA process. From this discussion it should be clear that the implicit assumption that all the parameters of the model are constant while we change the rule governing the money supply, is a strong assumption, particularly if we want to analyze long periods of time ^{1/}. Nevertheless, we assume that there will be a transition period, during which people will utilize something approximating the old process. That is, perhaps the past is full of promises and attempts from the government or monetary authorities that prices will remain stable or that the rate of inflation will be lower or that monetary emission will slow down. Moreover there are probably cases in changing the rule. Therefore people take some time in assuming that any change in the rule governing the money supply as a permanent one. It is precisely during this period that our simulations will be relevant.

The first simulation which is illustrated in Figure 1 shows the paths followed by the rate of change in nominal income, the rate of inflation and the rate of change in detrended output when

^{1/} Of course this might be a problem too for the stability of our estimates. If the ARIMA process is not stable neither can be the parameters of the transfer functions.

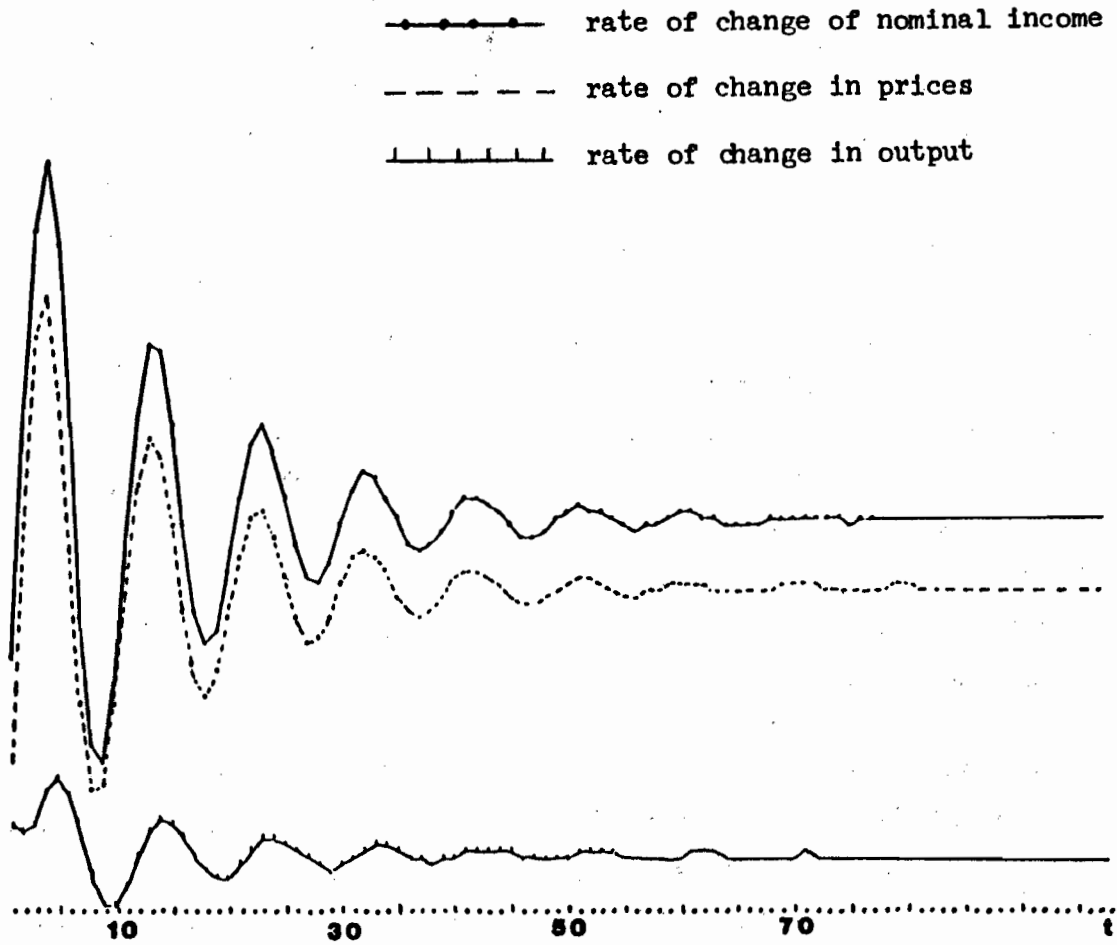


Fig. 1 --Simulation 1 for Argentina.

the money supply is shifted from a rate of growth of zero to a rate of growth of 10 per cent per period. The convergence to the new steady state is oscillatory for the three variables. Inflation accelerates during the first year reaching a peak at the end of the fourth quarter; during the second year inflation slows down and then accelerates again reaching a second peak at the 13th quarter. The rate of change in detrended output also accelerates during the first year but it peaks one period later than inflation, so during the first quarter of the second year output increases while the rate of inflation decreases. Similarly, the first trough of output is two periods later than the trough in the rate of inflation, so we observe inflation accelerating and output probably decreasing, a phenomenon known as stagflation. It should be noticed that from quarter 8 to 12 the rate of change in detrended output is negative so output will tend to be below the trend and consequently the unemployment rate above its natural level, while the rate of inflation is accelerating; this would be an illustration of a lower part of a counterclockwise loop in the conventional Phillips' Curve analysis. The other parts of the loop are readily observed in the following quarters as well as in the previous quarters ^{1/}.

The second simulation which is illustrated in Figure 2 considers a shift of the rate of growth of the money supply from zero to 10 per cent from period 1 to 30, and then it is shifted back to zero in period 31 and kept at that level thereafter. In this case we observe that the paths towards the final equilibrium level of the variables is oscillating and that a deep trough in the rate of inflation is reached 4 quarters after the reduction in the money supply while the trough in output is reached 5 periods after. One interesting aspect is illustrated in the four quarters between period 34 to period 37, during this quarters output is certainly below the trend and consequently we should expect a relatively high unemployment rate. At the

^{1/} There are two factors playing an important role in the determination of the loops. One is the lag structure in the transfer functions and the other is the autoregressive term in detrended income.

—•—•—•— rate of change of nominal income
- - - - - rate of change in prices
| | | | | rate of change in output

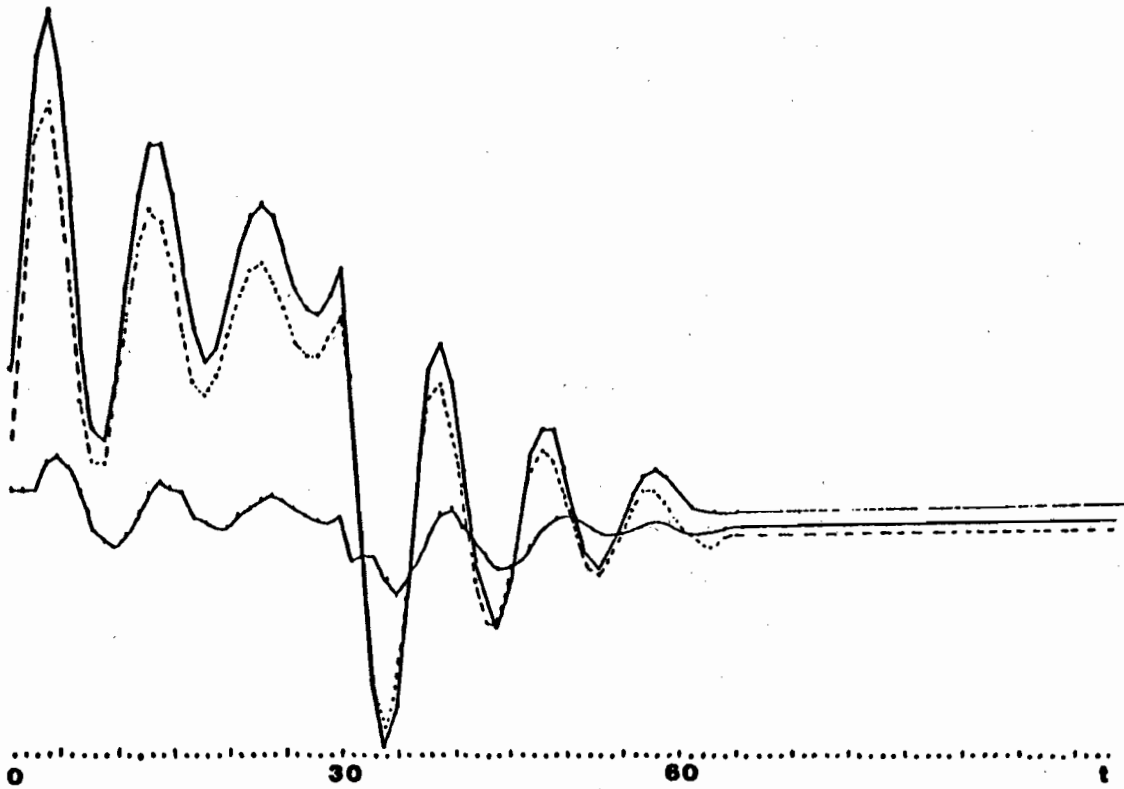


Fig. 2 --Simulation 2 for Argentina.

same time inflation is accelerating, this is a time when many people could think that the "old remedy to cure inflation does not work" because the reduction in the money supply not only have increased unemployment but also the rate of inflation is accelerating.

The third simulation is similar to the second but in place of an abrupt reduction in the money supply in period 31 we reduce the money supply to 8 per cent in the first year, to 6 per cent in the second year, and so on.

We observe that the fall in output is not as abrupt as it was in the previous case. In Figure 2 the trough in period 34 reached the value -3.4 per cent while in Figure 3 the trough in period 47 reached the value -1.8 per cent. Also it should be noticed that the convergence to the new steady state does not exhibit the large oscillations of the previous case; that is, in this case convergence is smoother.

Now Figures 4, 5 and 6 illustrate the same simulations for the case of Brazil. In simulation 1 (see Figure 4) we observe that the shift of the rate of growth of the money supply from 0 per cent to 10 per cent produces an initial overshoot of nominal income, prices and output but after a few oscillations they converge to their long run equilibrium values. We observe that nominal income and output reach a peak a quarter before prices; then during the fifth quarter we observe prices accelerating and output slowing down ^{1/}.

Simulation 2 for Brazil (see Figure 5) shows a big fall in output produced by an abrupt shift of monetary policy from a rate of growth of the money supply of 10 per cent to a rate of growth of 0 per cent.

Finally simulation 3 for Brazil illustrates the advantages of gradualism in stabilizing the economy (this same conclusion is reached by Goncalves (1974) for Brazil although in the context of a different

^{1/} It should be noticed the difference in the oscillatory pattern of nominal income between Brazil and Argentina. This is due to the different lag structure in the transfer functions for nominal income.

—•—•—•— rate of change of nominal income
- - - - - rate of change in prices
| | | | | rate of change in output

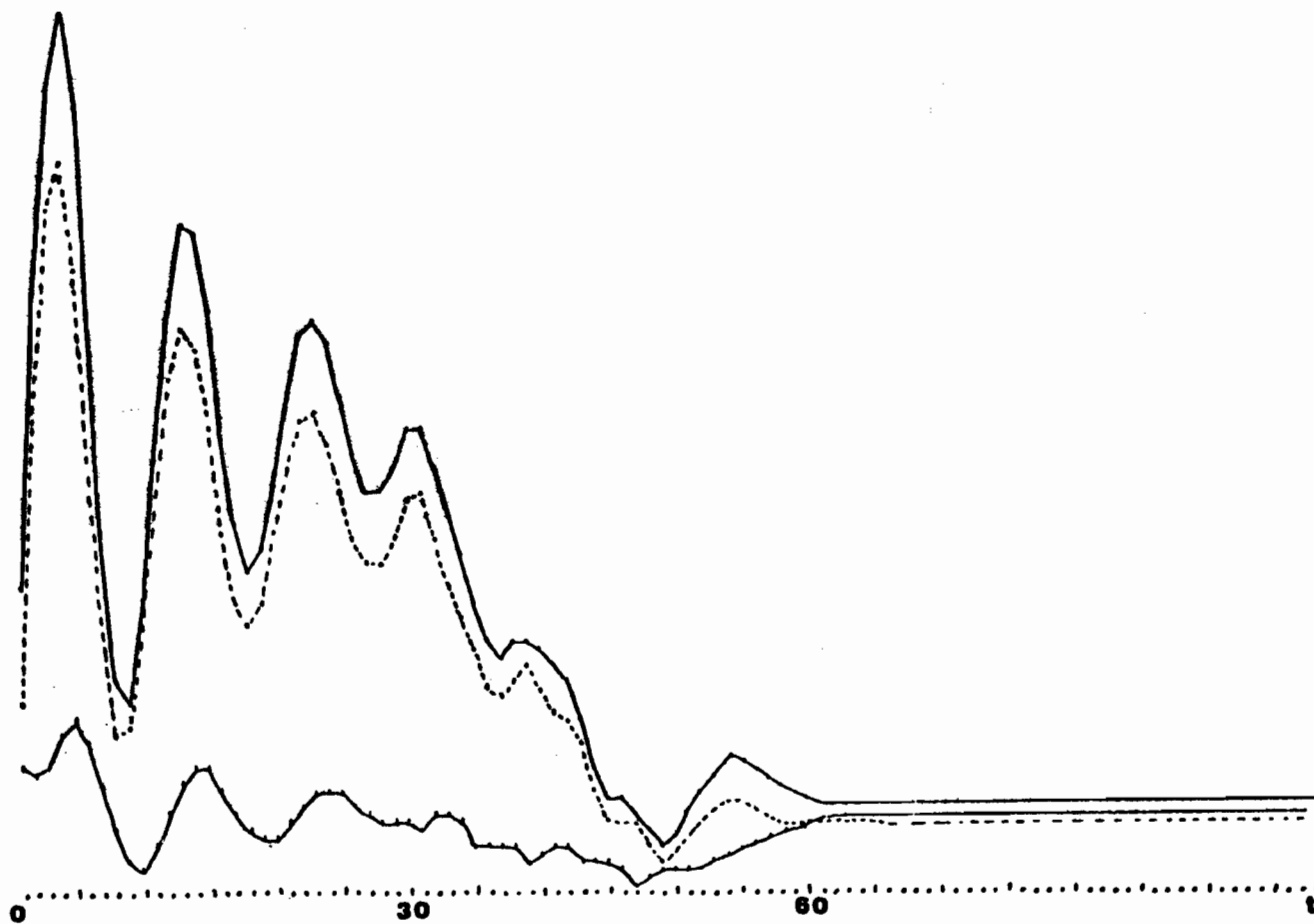
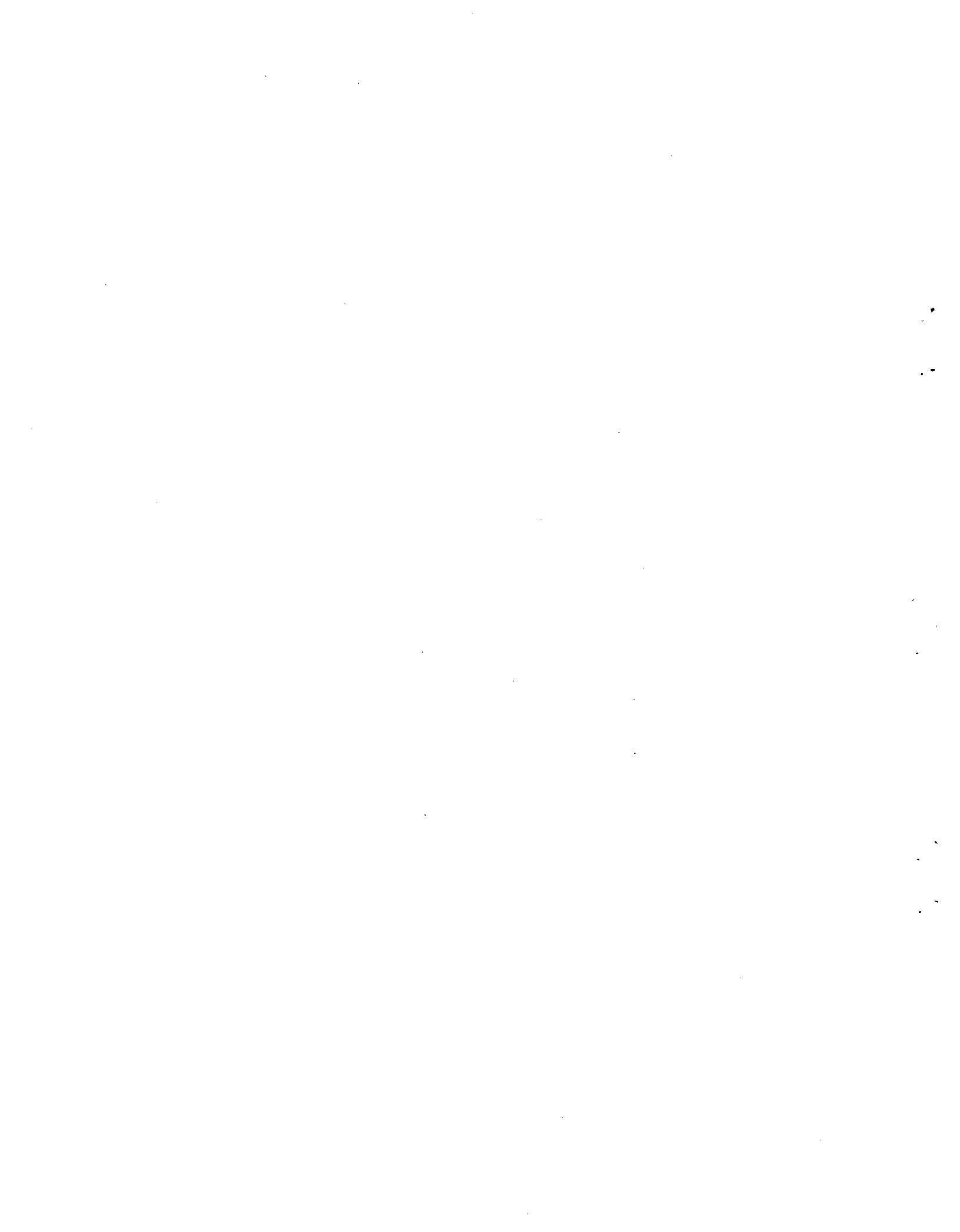


Fig. 3.--Simulation 3 for Argentina.



—•—•—•— rate of change of nominal income
- - - - - rate of change in prices
/ / / / / rate of change in output

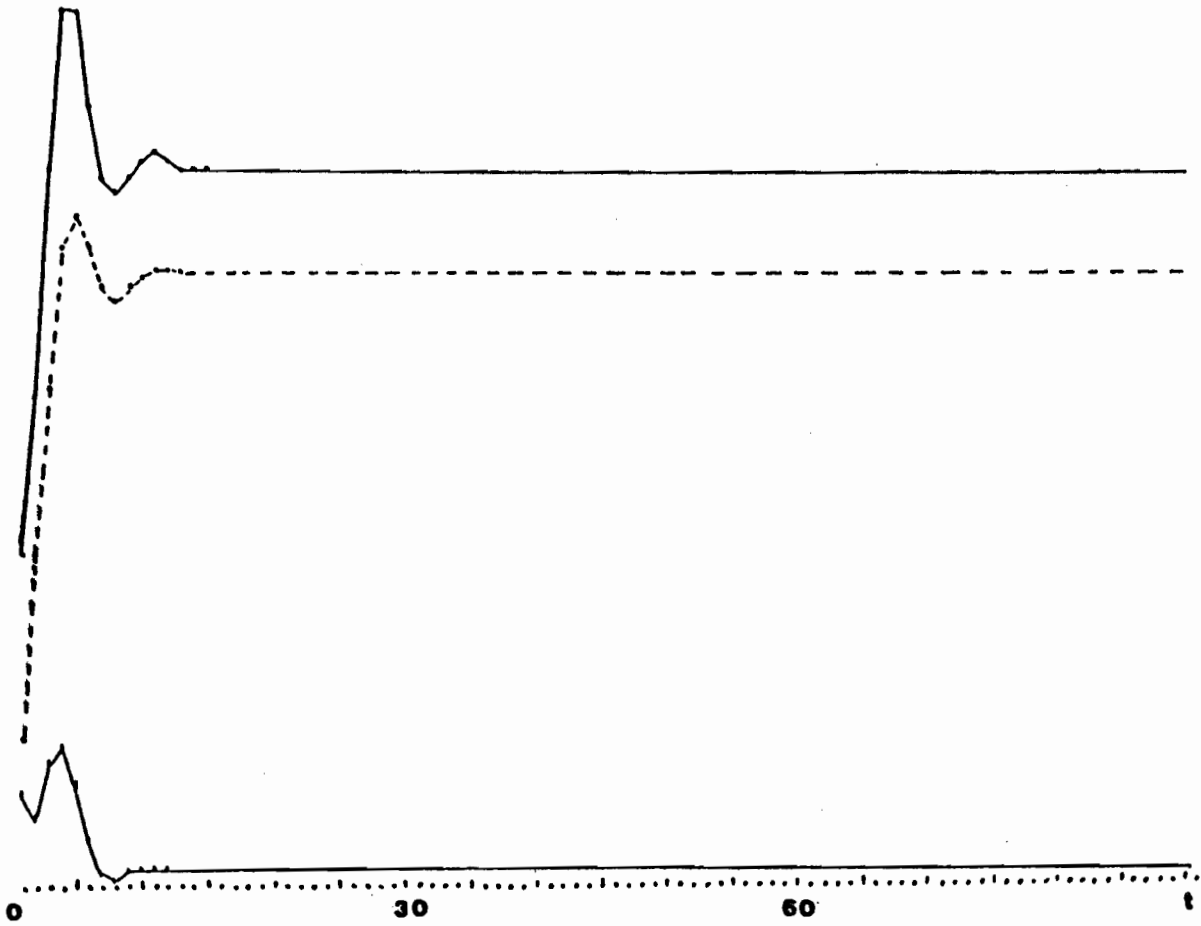
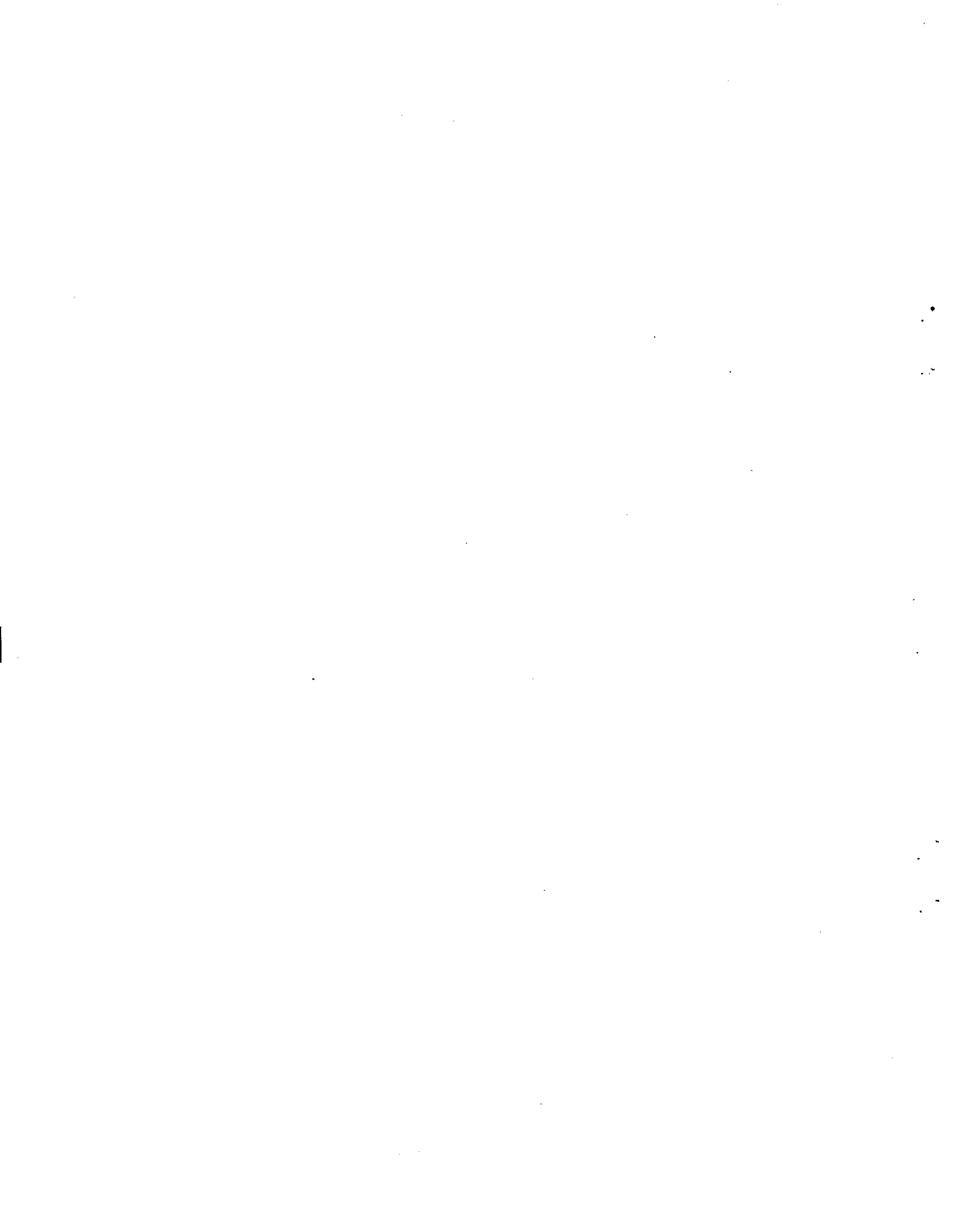


Fig. 4 .--Simulation 1 for Brazil.



—•—•—•— rate of change in nominal income
- - - - - rate of change in prices
| | | | | rate of change in output

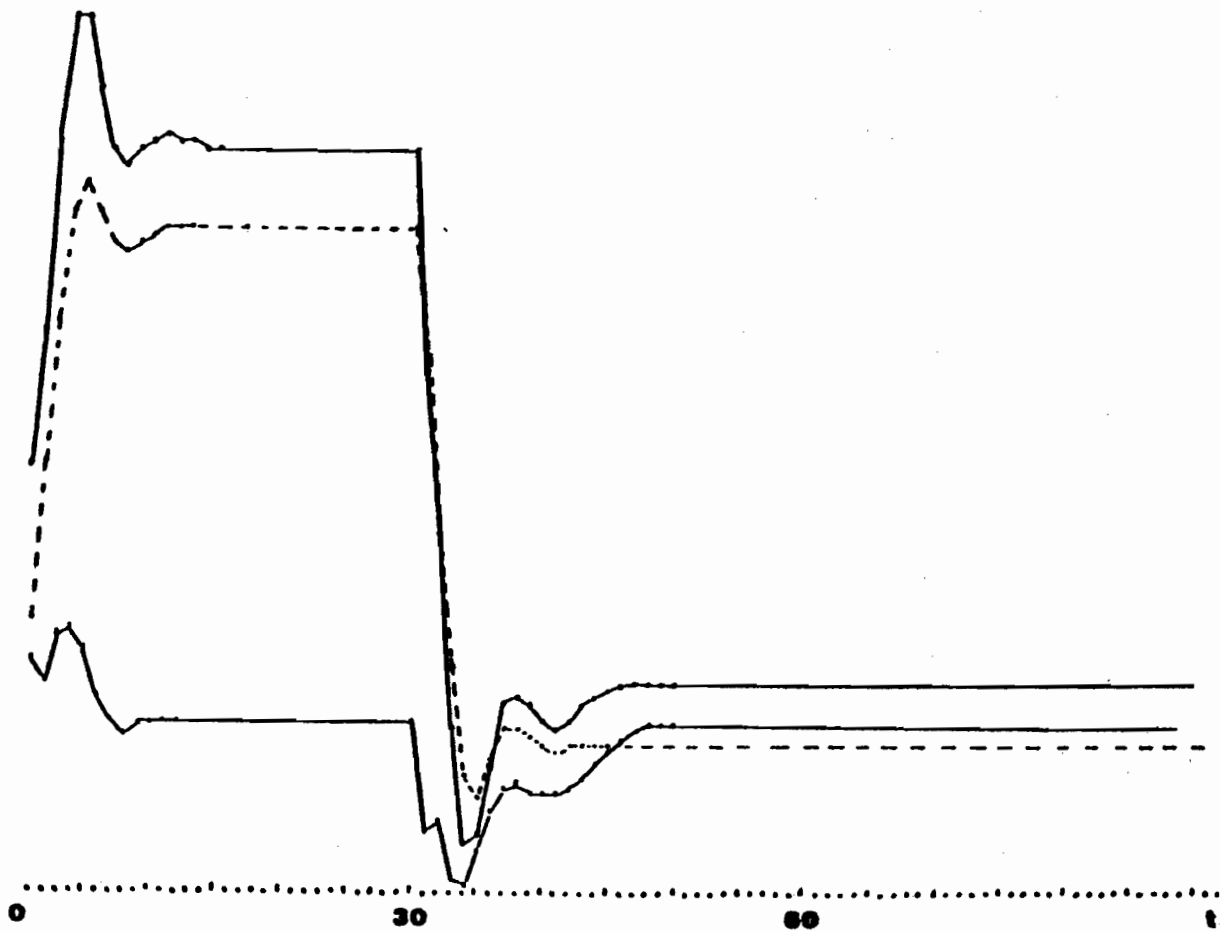


Fig. 5 .--Simulation 2 for Brazil.

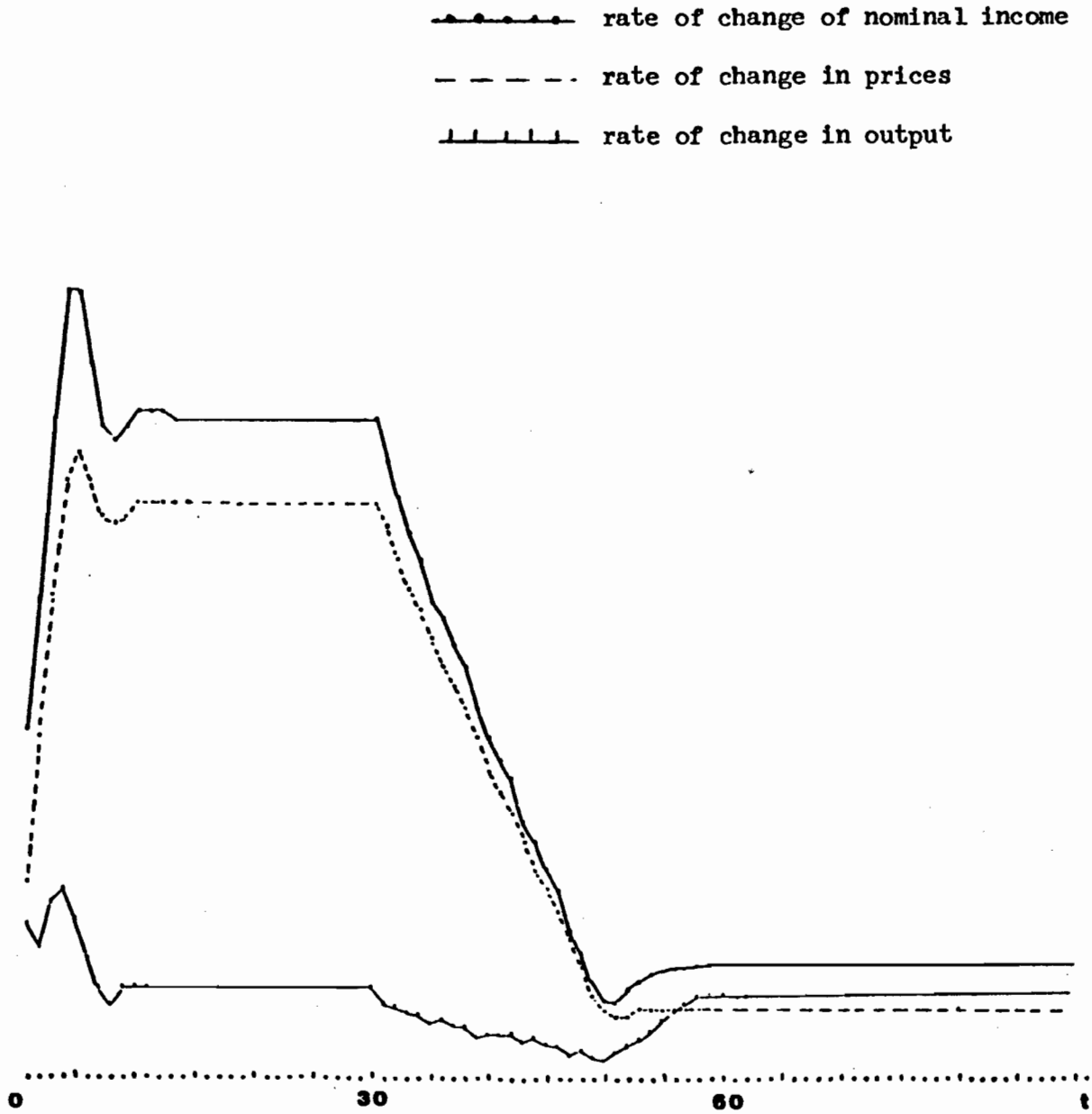


Fig. 6 .--Simulation 3 for Brazil.

model). We observe from Figure 6 that during the stabilization period the fall in output is smaller than in the previous simulation that assumed an abrupt change of monetary policy.

6. Conclusions

As indicated in the title of this paper we have tried to explain the short run dynamics of prices and output. An indicator of the degree to which this objective has been achieved could be the part of the variance in prices and output that has been explained by the model. In other words, we could look at the R^2 's obtained in our transfer functions or regressions. For the case of Argentina, the R^2 's for prices and nominal income have been close to .50 while for detrended output the R^2 's have been in the order of .35.

In the case of Brazil we obtained R^2 's around .45 for prices, .30 for nominal income and .43 for detrended real income.

Other indicators are the standard error of the estimates and the "t" values. Standard errors have been reported in the tables of section 4. Not all the estimates of the parameters are significantly different from zero at the 0.05 level but many of them are indeed significantly different from zero at the 0.05 level in a two-tailed test. Other estimates are small in absolute value and not significantly different from zero--for example, in the case of the estimates of $Dy_{c,t-1}$ and c in the transfer functions for prices and expected prices--however this is not in contrast with the theoretical model. As was mentioned above, these parameters can be close to zero. Finally, there are other parameters that have large standard errors, in particular the slope coefficient of our short run Phillips' curve, indicating that this relationship is empirically unstable.

In general the estimates for Argentina are more precise than the estimates obtained for Brazil. In both countries better fits were obtained for the rate of change in prices than for detrended income. The good performance of the model in explaining the rate of inflation can be illustrated by plotting the actual and fitted

/values from

values from the reduced form for prices. This is shown in Figures 7 and 8. Figure 7 shows the case for Argentina. Here we observe that the model behaves well in explaining inflation, and only in two observations--one near the beginning and one near the end of the period--the observed rate of inflation differs substantially from the fitted value.

Figure 8 illustrates the case of Brazil. Here we also observe the good performance of the model in explaining the large oscillations of the rate of inflation. Only in a few observations near the middle of the period do actual values differ substantially from the fitted values.

Although our results seem to be good relative to many other empirical studies working with highly noisy quarterly series, we still are not sure that we have really separated the true signal from the noise. That is, in explaining the movements of output out of its long run trend we have only used monetary shocks that impeded a correct anticipation of prices and in this sense people were surprised (or fooled) during short periods of time. As long as this is the only cause that produces cyclical fluctuations around the trend, then our model seems to behave well.

From a theoretical point of view we can say that our model makes use of two relatively new aspects of macroeconomic theory. One is the hypothesis of rational expectations and the other is a sort of Phillips equation playing the role of the "missing equation" which, according to Friedman (1971) states the difference between the quantity theory of money and the Keynesian income-expenditure theory.

The simulation analysis performed in Section 6 clearly illustrates many of the situations found in practice such as shifting short run Phillips curves, counterclockwise loops, and stagflation periods. They also illustrate the advantage of gradualism in stabilizing an economy. It was shown that an abrupt fall in the rate of growth of the money supply introduces a big oscillation in the system in the case of Argentina and a deep fall in output in both Argentina

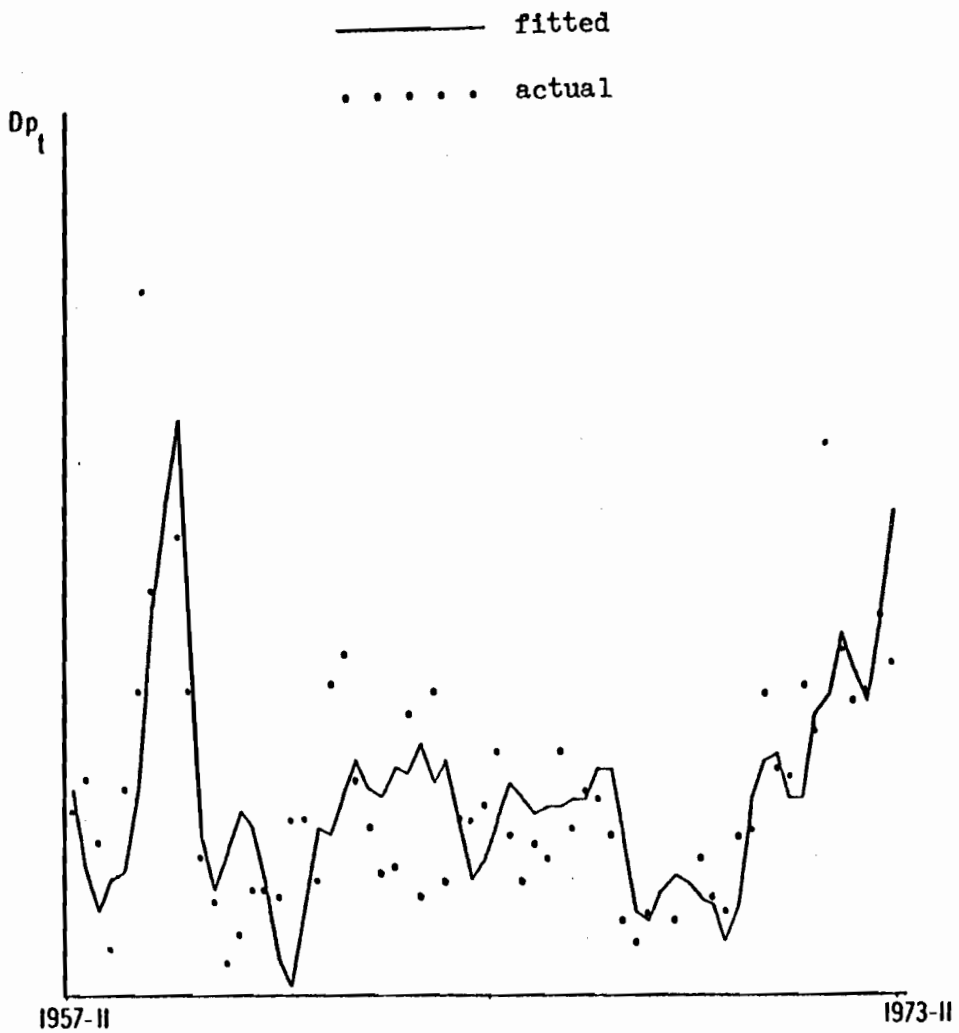


Fig. 7 --Actual and fitted values for the rate of change in prices. (Argentina)

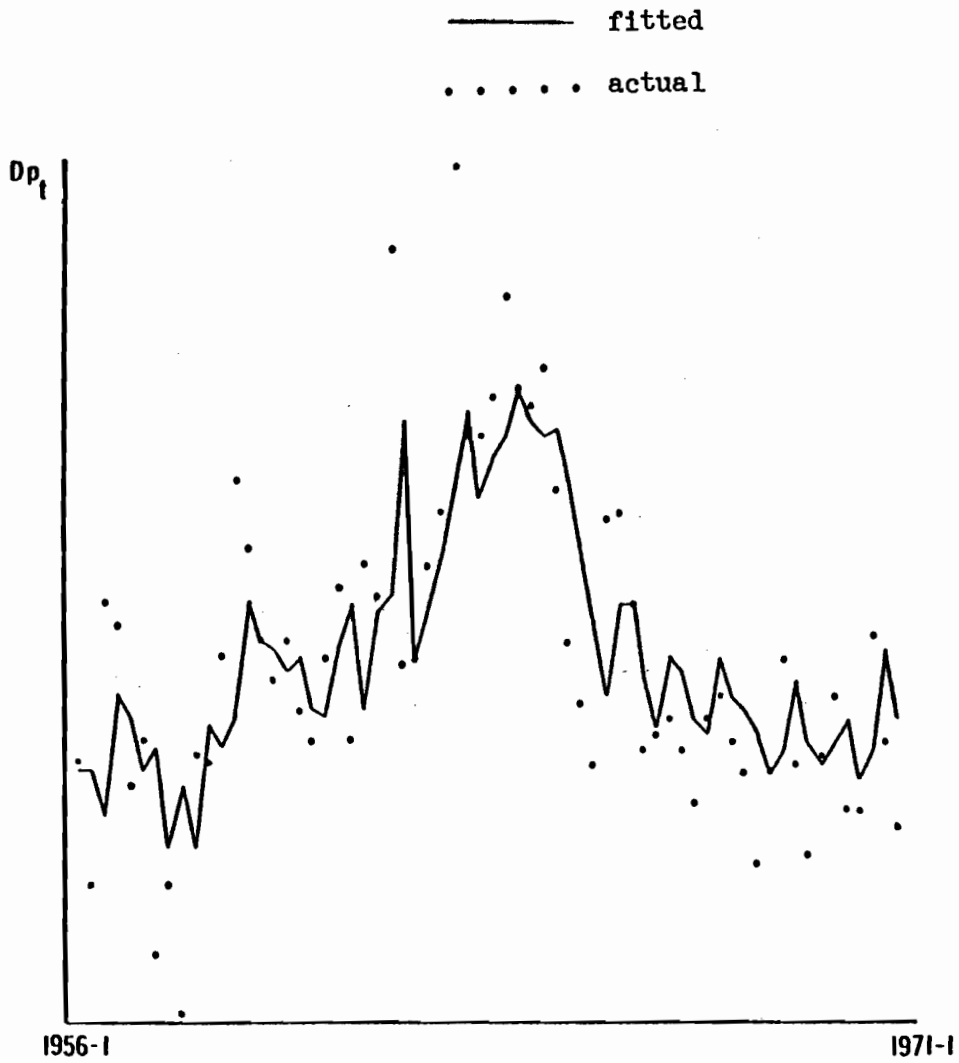
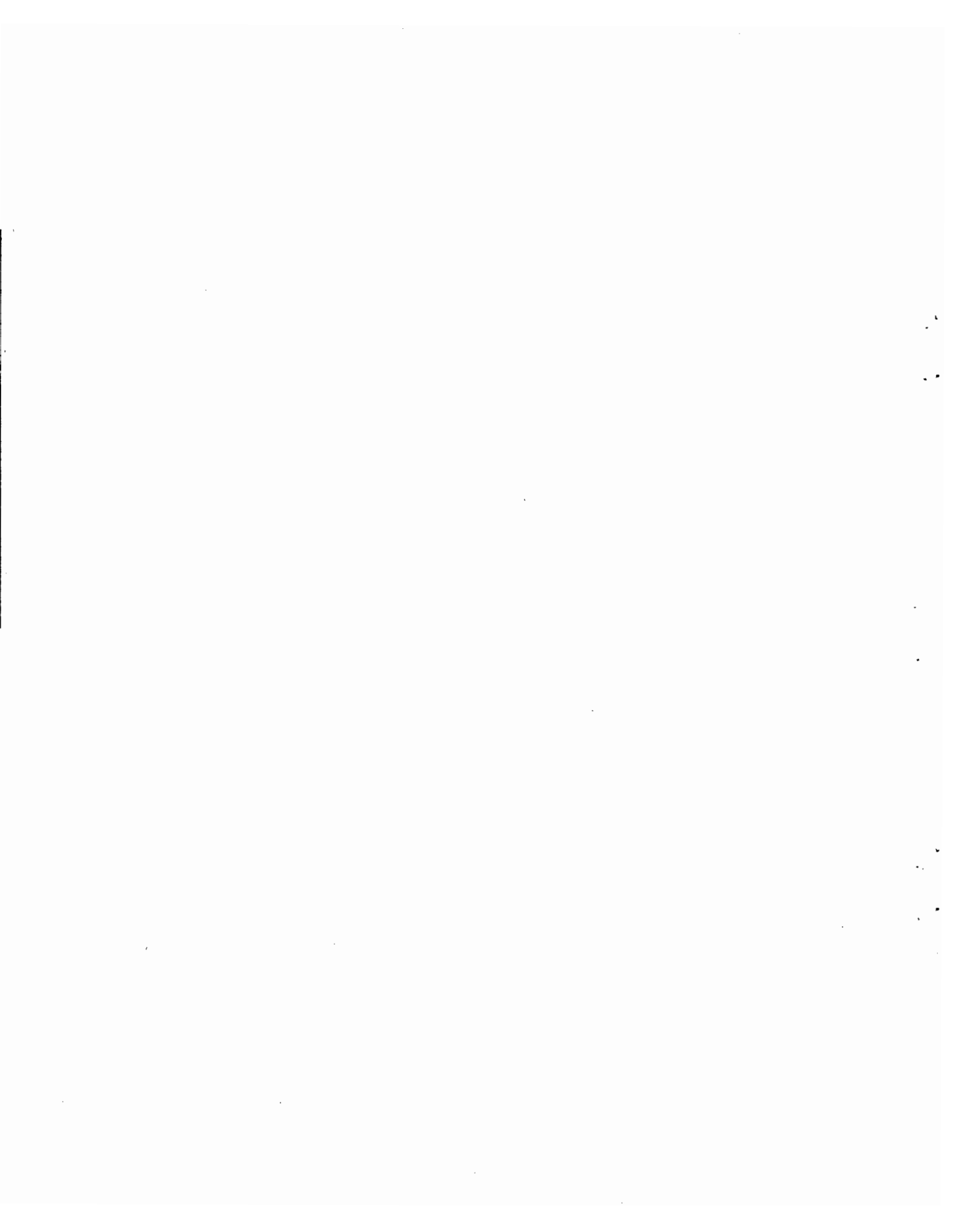


Fig. 8 .--Actual and fitted values for the rate of change in prices. (Brazil)



and Brazil. On the other hand, a gradual reduction in the rate of growth of the money supply produces a different effect. Firstly, no big oscillations are observed in the endogenous variables of the system. Secondly, the fall in output is not as large as it was in the previous case although the system reaches its new steady state in a period of time longer.

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APPENDIX A

THE ALGEBRA OF RATIONAL EXPECTATIONS

The methodology of this appendix is similar to the methodology developed by Sargent and Wallace [1975].

We start from the system (1) - (3) and we first solve equation (3) for r_t

$$r_t = b^{-1} \phi_{m,t} - b^{-1} p_t - b^{-1} y_t - b^{-1} u_{3t}$$

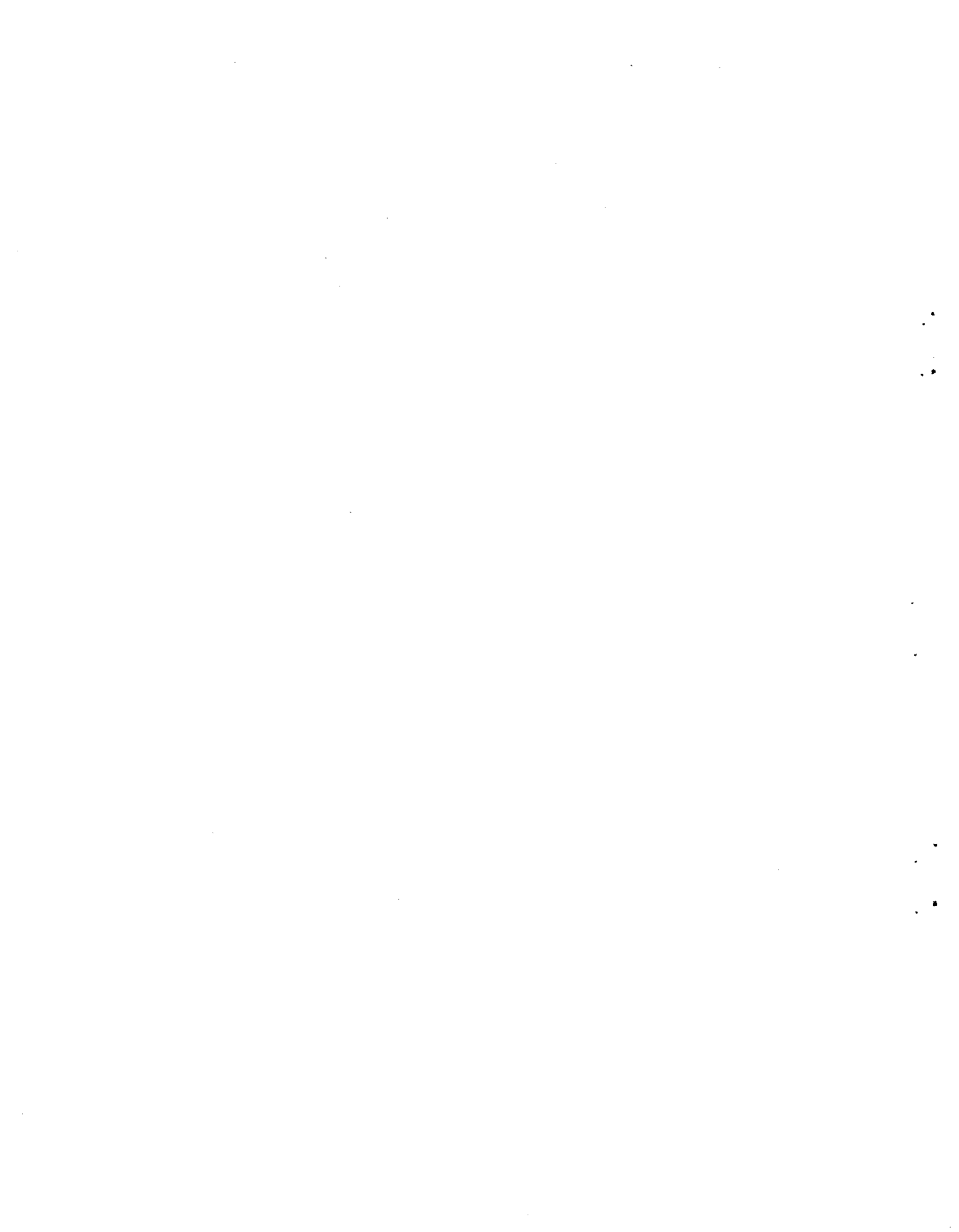
Substituting this last expression and (4) and (5) in (2) gives

$$y_t - y_{n,t} = g + cb^{-1} \phi_{m,t} - cb^{-1} p_t - cb^{-1} y_t - cb^{-1} u_{3t} - cE p_{t+1} + cE p_t + u_{2t}$$

Adding $(cb^{-1} y_{n,t} - cb^{-1} y_{n,t})$ in the above expression gives

$$y_t - y_{n,t} + cb^{-1} (y_t - y_{n,t}) = g + cb^{-1} \phi_{m,t} - cb^{-1} p_t - cE p_{t+1} + cE p_t - cb^{-1} y_{n,t} + u_{2t} - cb^{-1} u_{3t}$$

Then



$$\begin{aligned} y_t - y_{n,t} &= g/(1 + cb^{-1}) + [cb^{-1}/(1 + cb^{-1})]\phi_{m,t} - [cb^{-1}/(1 + cb^{-1})]p_t \\ &\quad - [c/(1 + cb^{-1})]Ep_{t+1} + [c/(1 + cb^{-1})]Ep_t \\ &\quad - [cb^{-1}/(1 + cb^{-1})]y_{n,t} + [1/(1 + cb^{-1})]u_{2,t} \\ &\quad - [cb^{-1}/(1 + cb^{-1})]u_{3t} \end{aligned} \tag{A.1}$$

Equating this last expression to (1) we have

$$\begin{aligned} ap_t - aEp_t + ky_{c,t-1} + u_{1t} &= g/(1 + cb^{-1}) + [cb^{-1}/(1 + cb^{-1})]\phi_{m,t} \\ &\quad - [cb^{-1}/(1 + cb^{-1})]p_t - [c/(1 + cb^{-1})]Ep_{t+1} + [c/(1 + cb^{-1})]Ep_t \\ &\quad - [cb^{-1}/(1 + cb^{-1})]y_{n,t} + [1/(1 + cb^{-1})]u_{2t} - [cb^{-1}/(1 + cb^{-1})]u_{3t} \end{aligned}$$

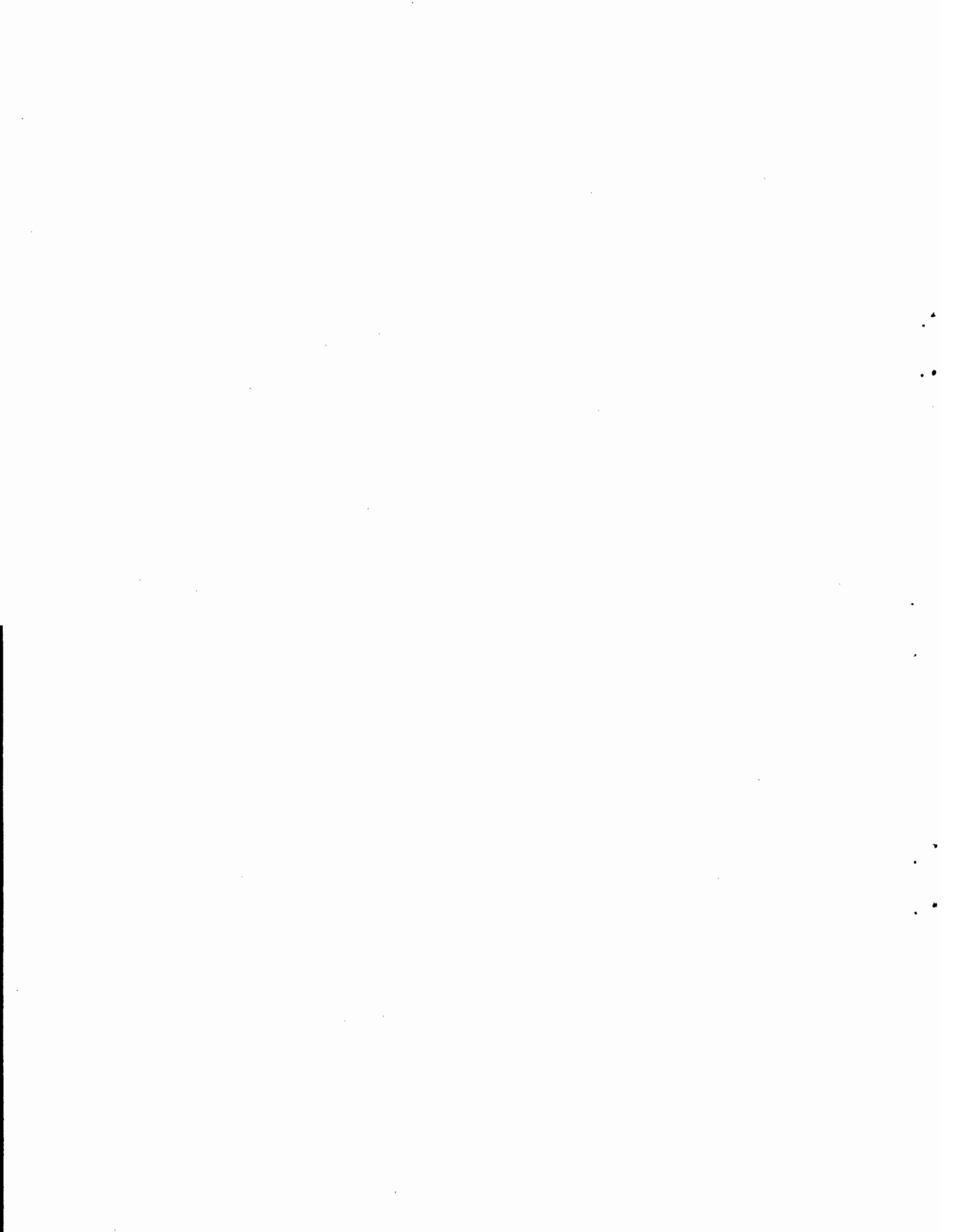
Solving the last expression for p_t the following expression is obtained

$$p_t = J_0 Ep_t + J_1 Ep_{t+1} + J_2 (\phi_{m,t} - y_{n,t}) + J_3 y_{c,t-1} + J_4 u_{1t} + J_5 u_{2t} + J_6 u_{3t} + J_7 \tag{A.2}$$

where

$$J_0 = [a + c/(1 + cb^{-1})]/\theta$$

$$J_1 = [-c/(1 + cb^{-1})]/\theta$$



$$J_2 = [cb^{-1}/(1 + cb^{-1})]/\theta$$

$$J_3 = -k/\theta$$

$$J_4 = -1/\theta$$

$$J_5 = [1/(1 + cb^{-1})]/\theta$$

$$J_6 = -J_2$$

$$J_7 = [g/(1 + cb^{-1})]/\theta$$

$$\theta = a + cb^{-1}/(1 + cb^{-1})$$

Equation (A.2) can be written more compactly as

$$p_t = J_0 E p_t + J_1 E p_{t+1} + N_t \tag{A.3}$$

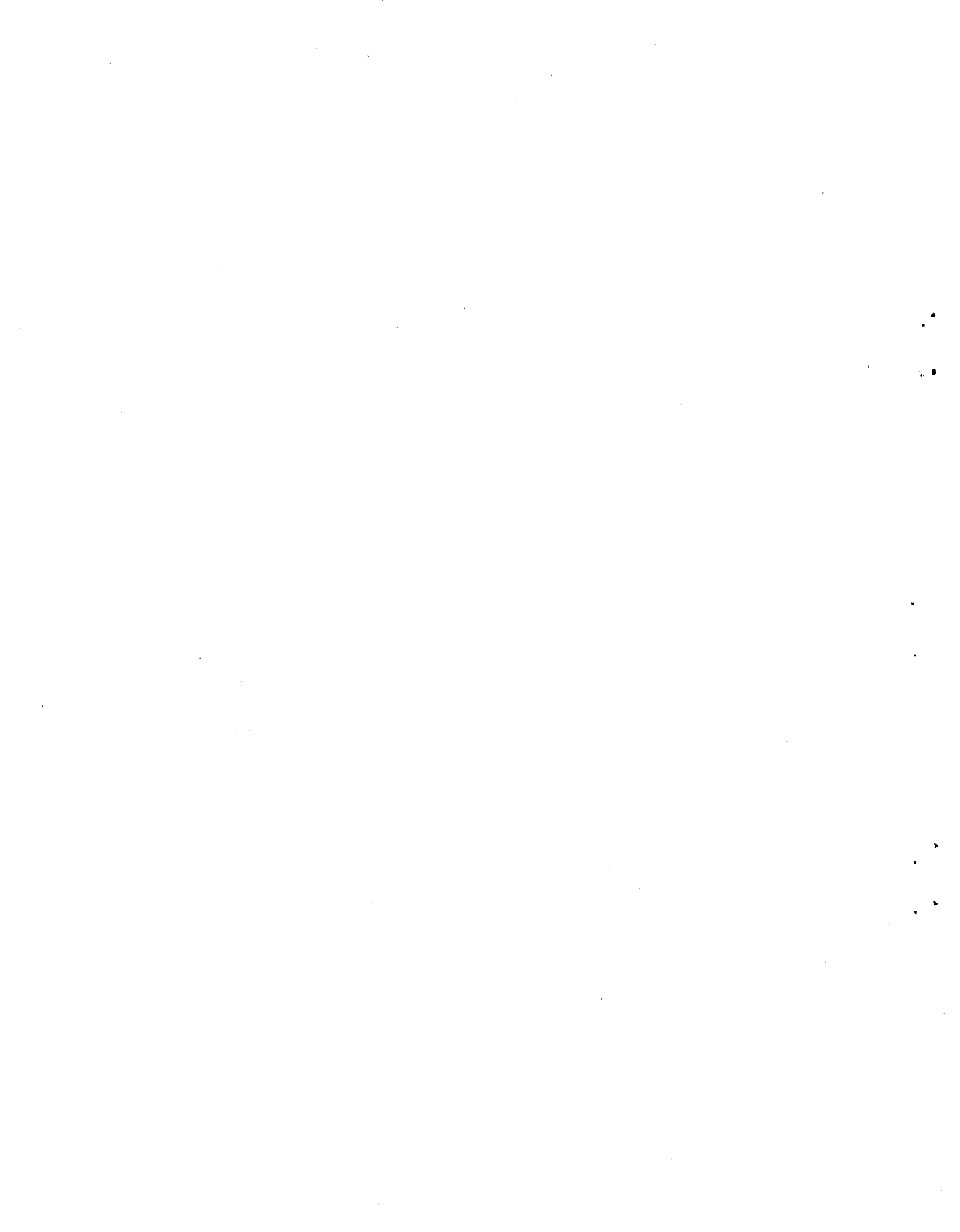
where

$$N_t = J_2 (\phi_{m,t} - y_{n,t}) + J_3 y_{c,t-1} + J_7 + w_t$$

where

$$w_t = J_4 u_{1t} + J_5 u_{2t} + J_6 u_{3t}$$

is a random variable normally distributed with zero mean.



Taking expectation in (A.3) conditional on the information available as of the end of period $t - 1$ the following expressions are obtained

$$E p_t = J_0 E p_t + J_1 E p_{t+1} + E N_t \quad (A.4)$$

and

$$E p_t = [J_1 / (1 - J_0)] E p_{t+1} + [1 / (1 - J_0)] E N_t \quad (A.5)$$

this last expression can be generalized to

$$E p_{t+j} = [J_1 / (1 - J_0)] E p_{t+j+1} + [1 / (1 - J_0)] E N_{t+j} \quad (A.6)$$

Repeatedly substituting (A.6) in (A.5), we obtain,

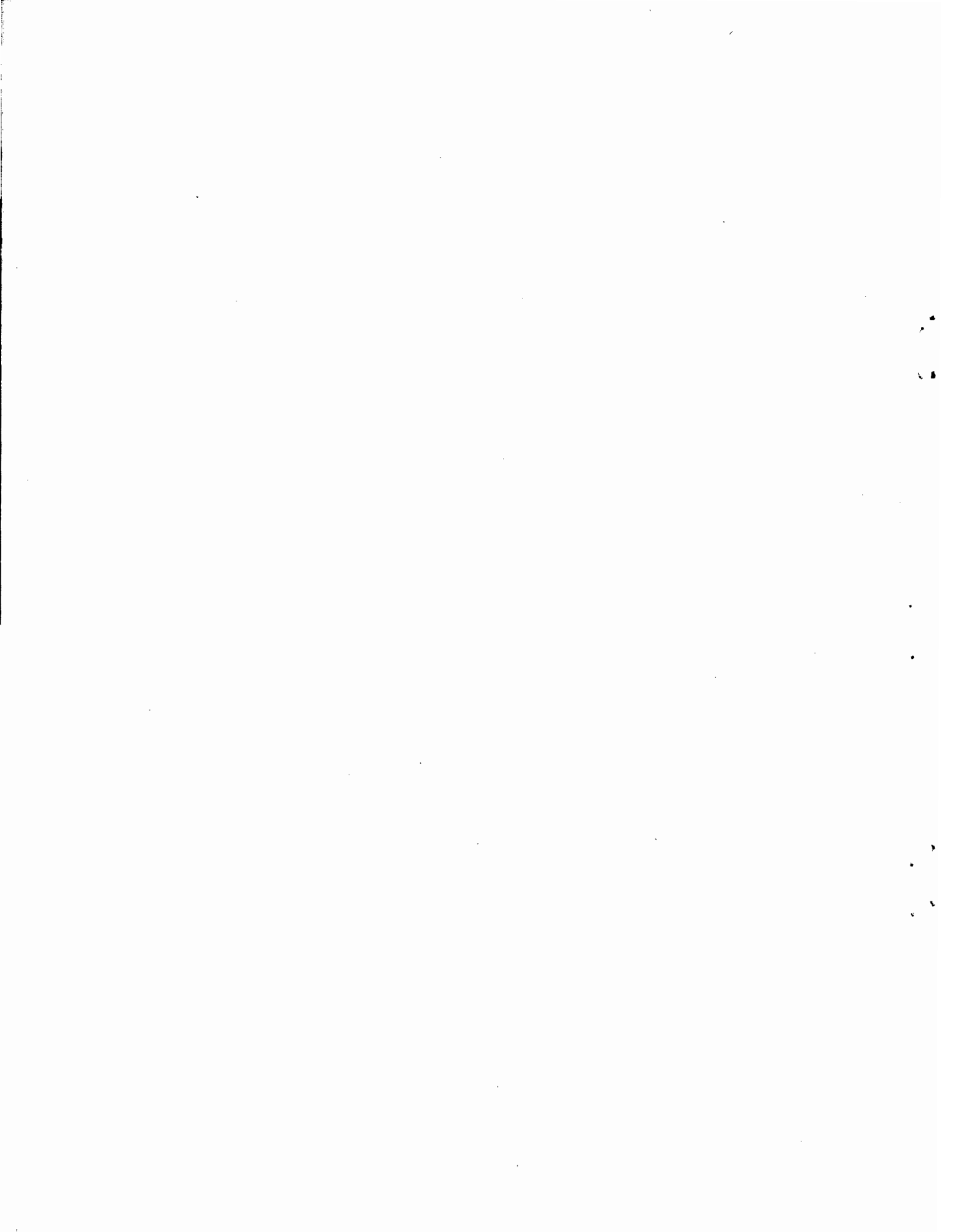
$$E p_t = [1 / (1 - J_0)] \sum_{j=0}^{\infty} [J_1 / (1 - J_0)]^j E N_{t+j} + [J_1 / (1 - J_0)]^{n+1} E p_{t+n+1} \quad (A.7)$$

Now, notice that $0 < J_1 / (1 - J_0) = 1 / (1 - b^{-1}) < 1$, then it is assumed that $\lim_{n \rightarrow \infty} [J_1 / (1 - J_0)]^{n+1} \approx 0$. Then the limit of (A.7) for n approaching infinite gives the following equation

$$E p_t = [1 / (1 - J_0)] \sum_{j=0}^{\infty} [J_1 / (1 - J_0)]^j E N_{t+j} \quad (A.8)$$

or, for period $t + 1$

$$E p_{t+1} = [1 / (1 - J_0)] \sum_{j=0}^{\infty} [J_1 / (1 - J_0)]^j E N_{t+j+1} \quad (A.9)$$



From equations (A.8) and (A.9) we notice that we should obtain some workable relationship for EN_{t+j} in order to get an expression representing the formation of expected prices. We know from (A.3) that

$$N_{t+j} = J_2(\phi_{t+j} - y_{n,t+j}) + J_3 y_{c,t+j-1} + J_7 + w_{t+j} \quad (A.10)$$

In order to apply the expectation operator above we need an assumption about the stochastic process followed by $y_{c,t}$. For simplicity I will assume that $y_{c,t}$ follows an autoregressive process

$$y_{c,t} = \alpha y_{c,t-1} + e_t$$

where e_t is an independently distributed random term with zero mean. This assumption is not restrictive; any other process in the class of ARIMA process can be assumed without loss of generality. It can be proved that the nature of the process for $y_{c,t}$ will be reflected in the autoregressive-moving average terms for $y_{c,t}$ in the transfer function

Applying the expectation operator in (A.10) we obtain

$$EN_{t+j} = J_2(E\phi_{t+j} - y_{n,t+j}) + J_3 \alpha^j y_{c,t-1} + J_7 \quad (A.11)$$

Substituting (A.11) in (A.8)-(A.9), and considering that $J_2/(1 - J_0) = 1/(1 - b)$ and that $J_1/(1 - J_0) = 1/(1 - b^{-1})$ we have



$$\begin{aligned} E p_t = & [1/(1 - b)] \sum_{j=0}^{\infty} [1/(1 - b^{-1})]^j [E \phi_{m,t+j} - y_{n,t+j}] \\ & + J_3/(1 - J_0) \sum_{j=0}^{\infty} [\alpha/(1 - b^{-1})]^j y_{c,t-1} + c_0 \end{aligned} \quad (A.12)$$

or

$$\begin{aligned} E p_{t+1} = & [1/(1 - b)] \sum_{j=0}^{\infty} [1/(1 - b^{-1})]^{j+1} [E \phi_{m,t+j+1} - y_{n,t+j+1}] \\ & + [J_3/(1 - J_0)] \sum_{j=0}^{\infty} [\alpha/(1 - b^{-1})]^{j+1} y_{c,t-1} + c_0 \end{aligned} \quad (A.13)$$

where

$$c_0 = [g/c(b^{-1} - 1)] \sum_{j=0}^{\infty} [1/(1 - b^{-1})]^j$$

It should be noticed that in equations (A.12) and (A.13) the term $(\alpha/(1 - b^{-1}))$ is lower than one so the coefficient of $y_{c,t-1}$ converges to a finite number.

Equations (A.12) and (A.13) correspond to equations (6) and (7) of Section 2.

